Documentation of the CWE FB MC solution as basis for the formal approval-request *(Brussels, 13 March 2015)*

Annex 16.20 Mitigation to curtailment of price taking orders

Executive summary

CWE NRAs requested the CWE project to suggest a mitigation measure that prevents price taking orders (orders submitted at the price bounds set by the exchanges) to be curtailed because of "flow factor competition". Furthermore they challenged the project to find a solution that allowed some of the curtailment sharing principles that exist under ATC to be reflected in the mitigation proposal too.

As a mitigation option the project considered solutions that require a patch to the Euphemia. Several approaches were studied, which are presented in this document, together with some analysis on their impact. All considered options manage to absolve price taking orders from "flow factor competition", but differ in their ability to allow for the sharing of curtailments. The study reveals that sharing of curtailments in a flow based context immediately implies increasing the overall curtailment. However to meet the regulators request the project considers this sufficient grounds to accept the impact of the increased curtailment.

Finally the project proposes one of the investigated solutions as the mitigation option to be implemented. The proposed option manages to share curtailments between markets in a way that closely mimics the logic that currently exists under ATC.

Contents

1 Introduction

CWE NRAs requested the project to study the link between FBMC and short term generation adequacy. This study[1] focussed on order curtailment situations: situations where a market clears at maximum price, and can only fill part of the demand orders. The rationale being that order curtailment in DA is indicative for potential adequacy issues in real time. The study revealed a conceptual issue where order curtailment situations could be aggravated because non-price taking orders of adjacent markets would be favoured over price taking orders in the curtailed market. This situation is created due to the notion of "flow factor competition" that is intrinsic to FBMC.

The study did not manage to quantify this risk, but did qualitatively assert its existence. It further managed to illustrate the occurrence of this situation in Belgium under strong assumptions on prices in France. NRAs concluded that the existence of the risk was sufficient to ask for a mitigation measure. This report describes and analyses the suggested mitigation.

[1] http://www.casc.eu/media/Adequacy_study_report_CWE.pdf

2 Problem definition

2.1 Flow factor competition

Under FBMC there exists "flow factor competition": if two possible deals generate the same welfare, the one having the lowest impact on the scarce capacity will be selected. It also means that, in order to optimize the use of the grid and to maximize the market welfare, some sell (/buy) bids with lower (/higher) prices than other sell (/buy) bids will not systematically be selected with Flow-Based allocation. This is a well-known and intrinsic property of Flow-Based sometimes referred to as "flow factor competition".

2.2 Flow factor competition and price taking orders

Under normal circumstances this property of "flow factor competition" is accepted. However for the special case where the situation is exceptionally stressed because of scarcity in one particular zone, this "flow factor competition" could lead to order curtailment. It means that some buyers ready to pay any price (but unable to properly express this due to the price caps in the DA market) to import the energy would be rejected while lower buy bids in other bidding areas are selected. Some "price-taking orders" (buy orders capped at maximum price[2]) in one bidding area are not selected while lower buy orders in other bidding areas are. This would lead to the difficult situation where one bidding area is curtailed while the clearing prices in the other bidding areas are lower. This is the problematic situation we seek to mitigate.

[2] Note that price taking block orders are not considered price-taking orders in the curtailment context, since they impose a fill-or-kill constraint implying curtailment (partial rejection) is not permitted

2.3 Complication

The situation becomes more complex when two or more markets are simultaneously in curtailment. Is "flow factor competition" between price-taking orders from different markets acceptable, or should there be some mechanism in place to "fairly" distribute the curtailments across the involved markets? If a more "fair" distribution is required, the ambition is to identify a solution that mimics the curtailment sharing approach that exists today, where curtailment ratios are equalized as much as possible.

In the next section we suggest solutions for either approach.

3 Solution description

A solution is foreseen where the flow factor competition for price taking orders is absolved by treating price taking orders differently than non-price-taking orders. This solution should then be implemented in the market coupling algorithm. We will explore the solution in more detail in this section.

For the avoidance of doubt

Note that all solutions presented in this section implement a "penalty". Please mind that the penalties mentioned here are a modelling trick, where undesired results (curtailment of price taking orders) is made less desirable for the algorithm (via the penalty). They do not involve imposing penalties on any of the actors involved in the DA markets.

3.1 Solution 1: penalizing the non-acceptance of price taking orders

The issue that was identified in the initial adequacy study was that it could happen that some price taking orders could not be matched, because orders from a different market, not submitted at maximum price were accepted instead, due to a more favourable flow factor. The principle issue here is the capping of the price at which the price taking orders are submitted: they are treated as if they were willing only to pay the maximum price, whereas they could represent a larger willingness to pay.

To better reflect this higher willingness to pay, we want the algorithm to prefer solutions where price taking orders are not curtailed over solutions where price taking orders are curtailed. To achieve this goal, a penalty is added to the objective function:

Old objective	New objective
max welfare	**max** welfare – M * rejected PTO volume

Where PTO volume is the volume of the price taking orders, and M is a large value representing the penalty for failing to fill the price taking orders.

Interpretation

The solution to add a penalty to the objective function is a rather abstract formulation of the mitigation measure. However one can prove that this solution is mathematically equivalent with raising the price of the price taking orders by an amount of M, which might be a more intuitive way to think about the solution: raising the price of these orders is an effective means to balance the competition, even in case of highly unfavourable flow factors, back in favour of those orders submitted at 3000€/MWh.

3.2 Solution 2: sharing the curtailments

In the previous section we described a solution that effectively balances the competition between price taking orders and non-price taking orders in favour of the price taking ones. What it does not consider is the case where two markets are simultaneously in curtailment. Since the solution effectively raises the price of PTOs in both markets to the same value big M, what remains is the original "flow factor competition" problem. The result will again be that the market with the most favourable flow factor will receive the import.

Therefore in this section we consider a small evolution from the previous solution where we try to prevent "flow factor competition", even for the case of simultaneous curtailments. To this end we further update the new objective to factor in a quadratic component: rather than penalizing just the rejected PTO volume we consider a penalty involving a quadratic function of the rejected PTO volume: the penalty grows more quickly with increased curtailment, hence equilibrium can be expected where curtailments are roughly equal. This is not unlike the current approach used under ATC, which aims to equalize curtailment ratios, by minimizing a quadratic function involving the rejected PTO volume:

Linear objective	Quadratic objective
max welfare – M * rejected PTO volume	**max** welfare – M * (rejected PTO volume)^2

Mind that unless the price reaches 3000€/MWh the rejected PTO volume is zero, hence the penalty will be zero too. We should precise that under ATC this is done as a post-processing step, since it can be done without affecting the welfare. To try equalizing curtailments under FB, it does affect welfare and therefore it is considered in the first step of the algorithm by making changes to the objective function. Furthermore, sharing curtailments under FB – other than under ATC – increases the total amount of curtailment and thereby potentially affects system adequacy (Cf. section 4.3.3).

3.3 Solution 3: sharing the curtailments – alternative implementation

The updated quadratic objective penalizes the square of the rejected PTO volume. For a sufficiently large M this means that the objective becomes to minimize this penalty function. One can prove that optimal solutions try to equalize the curtailments in the different areas. However, if we compare this with the ATC curtailment logic, we see an inconsistency: for ATC the logic was to equalize curtailment ratios rather than curtailment volumes. The rationale being that since PXs pro-rate the curtailments across the orders submitted at maximum price for each of the hubs, equalizing the curtailment ratios between areas extends this logic across borders.

To remedy this oversight in solution 2 one could consider an alternative penalty:

Quadratic objective – solution 2	Quadratic objective – solution 3
max welfare – M * (rejected PTO volume)^2	**max** welfare – M * PTO volume * (rejected PTO ratio)^2

This new penalty in fact is identical to the way Euphemia manages curtailment today: a volume problem is solved, where this penalty (without the big M) is minimized, subject to all network constraints.

3.4 Issues related to the suggested implementation

There are some existing requirements in the Euphemia algorithm that under very specific circumstances can be impacted as a consequence of adding a penalty function to the objective. The impacted requirements are:

Line losses

Euphemia models line losses and requires that solutions should respect not only the physical but also financial losses: a flow should only be scheduled if the price difference between the areas is sufficiently large to offset the cost of the lost energy.

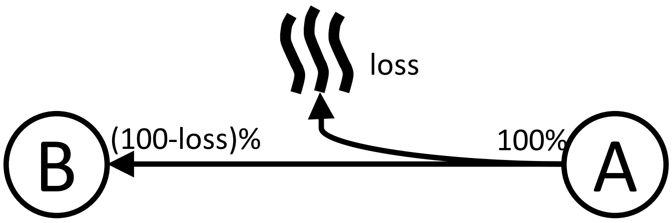
If we consider solution 1, which can be interpreted as treating PTOs algorithmically as if they are priced at a very high price (say 1M€), it becomes apparent that if the exporting market has a very high price too, but which is just shy of 3000€/MWh, then the financial losses requirement no longer will be respected. For the 1M€ PTO Euphemia "sees" an opportunity to create welfare and schedules the flow. After truncating the price to 3000€ the financial losses are no longer covered. Figure 1 illustrates this by example.

For the solutions 2 and 3 a similar logic can be applied: Euphemia still perceives an opportunity to schedule the flow. Not because it increases welfare, but rather because it avoids a penalty. The net effect obviously is the same.

Likelihood

For this scenario to materialize, we need:

- Order curtailment in the importing area;
- An adjacent area which is connected by a line that models losses;
- The price of this adjacent area to exceed 3000 / (1-loss), e.g. 2880€/MWh for a 4% loss.



$CR^{4\%}_{1M€} =$	(1-loss) x 1000MW x 1M€ = 9.6G€	—	2999€ x 1000MW =2.999M€	>>0
$CR^{4\%}_{3000€} =$	(1-loss) x 1000MW x 3000€ =2.88M€	—	2999€ x 1000MW =2.999M€	=-119k€ < 0

Figure 1 Illustration of modelling cable losses. The results below the figure correspond to 4% losses assuming 1000MW capacity. The first row is congestion rent assuming the 1M€ cap remains unaltered, whereas the second row illustrates the actual congestion rent, after the 3000€ cap price is applied.

Ramping

The ramping requirement limits the hour-to-hour change of a line flow. We consider the illustration from Figure 2 where we see the flow (grey bars) against time. Much like the losses example we can imagine a situation where one market is in curtailment (market B), and the other is just shy of being in curtailment. Euphemia schedules a flow from A to B to realize the welfare following from the perceived 1M€ price in B. If this situation occurs in period t, the ramping limit prohibits the flow in period t+1 to switch direction (provided the ramping limit is smaller than the capacity).

If the market situation for period t+1 would be such that market A has a lower price than market B, this would result in an adverse flow. This is only allowed, if the welfare gain in period t outweighs the welfare loss of period t+1. If the (negative) spread for period t+1 is larger than the spread for period t (after capping the price in B to 3000€/MWh), this property is violated.

Likelihood

For this scenario to materialize, we need:

- Order curtailment for only 1 market in period t;
- Flip of market direction for either period t+1 or t-1;
- The adverse price spread of period t+1 (or t-1) should be larger than the positive price spread for period t.

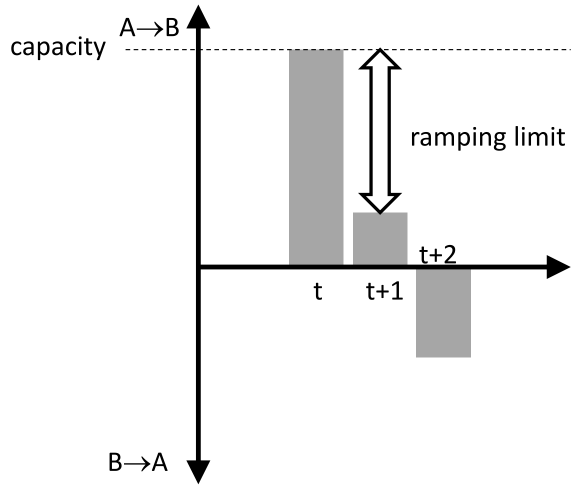


Figure 2 illustration of ramping limit

Negative congestion rent under FB "plain"

Under FB "intuitive" we know there should be a decomposition of the net positions into bilateral exchanges such that these exchanges go from a low price area to a high price area. Therefore by construction congestion rent is non-negative.

For FB "plain" a similar result exists and one can prove that the overall congestion rent is non-negative. Overall congestion rent can be considered the gain from selling the energy in importing areas minus the cost of buying this energy in the low priced area. If however Euphemia considers a huge gain in a curtailed market (the perceived price is 1M€), this would allow for exports from relatively high priced markets. After the price is capped to 3000€/MWh examples exist for which the overall congestion rent becomes negative.

Example

Imagine three markets with orders:

Market A	Buy 10@"1M€"
Market B	Buy 100@-2000
Market C	Sell 100@2000

FB constraints:

PTDF = [0, 0.6, 0.5];

RAM = 4

The optimal solution will try to accept all buy in market A. This energy needs to come from market C, but only RAM/($PTDF_C$-$PTDF_A$) = 4/0.5 = 8MW can be exchanged.

Under FB "plain" we can also schedule some energy from market C to market B (non-intuitively) to relieve a congestion. We schedule 10MW. This relieves the constraint by 10 * (0.5-0.6) = 1MW, allowing an additional 2 MW to be scheduled from C to A. The resulting net positions become:

[-10; -10; 20];

The corresponding clearing prices are:

Market A: "1M€"

Market B: -2000€

Market C: 2000€

And the total congestion rent = 1M€ x 10 + (-2000€) x 10 – 2000€ x 10 = 9.96M€

However after capping the price in market A to 3000€ this becomes:

CR = 3000€ x 10 + (-2000€) x 10 – 2000€ x 10 = -10k€, i.e. negative congestion rent.

4 Analysis

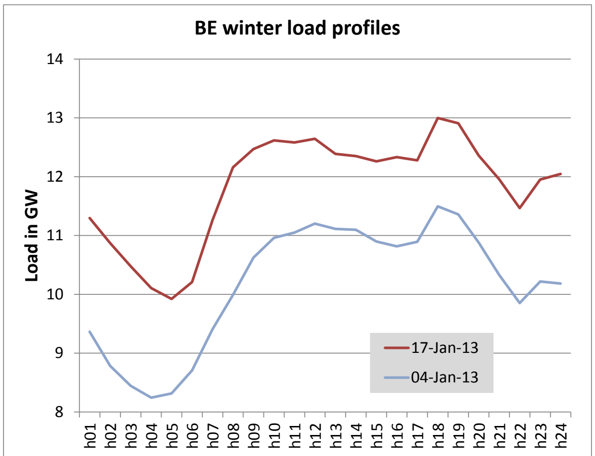
4.1 Prototype

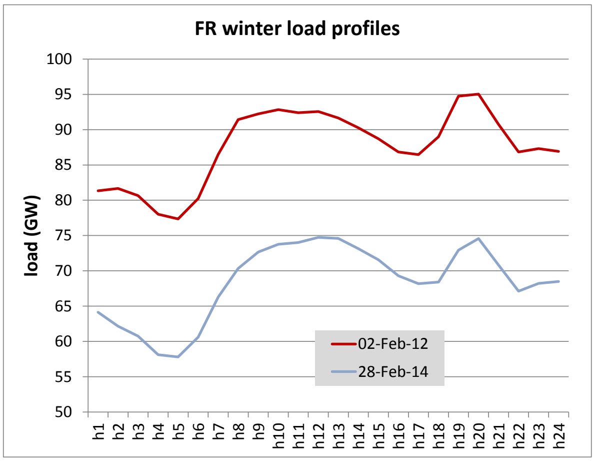
To allow a preliminary assessment of the efficacy of a proposed mitigation, the solutions 1, 2 and 3 from the previous section have been implemented in a prototype version of the algorithm. Simulation results are presented in this section.

4.2 Data

To assess the effectiveness on a more quantitative basis, we considered 2013 data where additional load was added to the order books of BE and FR. To have somewhat realistic assumption of the additional load that would be added to the order books, we considered historical load data. For the winter months (Jan, Feb, Dec) of the years 2011, 2012, 2013 and 2014 we considered the load for the hours 8-20. On the basis of this load data we ranked the days, and we considered high load winter days (98^{th} percentile) and typical load winter days (50^{th} percentile). Using this definition the following days were used to reference load:

	Typical load day	High load day
BE	4 Jan 2013	17 Jan 2013
FR	28 Feb 2014	2 Feb 2012





The average difference between the high and typical scenarios has been added to the historical demand (BE: 1654MWh; FR: 19131MWh).

Finally for the FR market an update was made for the borders with Switzerland, Spain, Italy and GB. If we assume high load in FR, we can assume these borders to be scheduled towards FR. We update OBKs to offset the historically scheduled flows and instead assume full FR imports.

For the FB parameters we used the data from the 2013 parallel run.

In the next section the results of the simulations are presented. Under the assumptions we made there were relatively little BE curtailments, and hence little data to study the double curtailment case. Therefore additional runs were made, with BE OBKs amended with respectively 500MW and 1000MW extra load.

Bias in dataset

Note that for both BE and FR we added a fixed value of load to the order books. The fixed value corresponds to the difference between a high load and a typical load day. Underlying this approach there implicitly is the assumption that all the days we consider can be regarded as typical days.

However when doing the analysis we focus on those cases where the price spiked to 3000€/MWh and these days correlate to already higher than average load. E.g. looking at days where FR prices spiked to 3000€/MWh in our dataset the average FR historical load was approximately 77GW, whereas the typical load day corresponded to 68GW. For BE the average historical load was approximately 15.6GW for 3000€/MWh hours vs 10.1GW typically.

4.3 Results

4.3.1 Single curtailment cases

This section explores those simulation results for which one market cleared at 3000€/MWh, whereas all other hubs cleared at lower prices. We focus on cases that resulted in a 3000€/MWh price without applying any patch. Mind that applying the patch will make it less beneficial to send energy to the market with the more favourable flow factor. Consequently this market might see its price rise to the 3000 level too. In case we apply the quadratic penalty, we may even expect energy to be curtailed in this market too.

To assess the effectiveness of the patch, we consider a situation where one of the markets cleared at 3000€/MWh. We focus on the FBI results where it was the French market that cleared at 3000€/MWh. We should expect curtailments to be lower after applying the patch.

Figure 3 shows the curtailments (in MW) for BE and FR. We compare the different algorithmic options:

- None: no Euphemia patch;
- Linear: the linear penalty;
- Quadratic: the quadratic penalty;
- Ratio: the quadratic penalty for curtailment ratios.

If we consider the left-most figure we see the results using the linear penalty function. The horizontal axis gives the curtailments without any patch, the vertical axis the curtailments after applying the patch. BE curtailments remain at 0, i.e. the patch does not trigger BE curtailments. However the patch does manage to relieve the FR curtailments as indicated by the data points moving below the 45 degree line.

The figure in the middle illustrates the effects of applying the quadratic penalty. It more strongly relieves the FR curtailments, and now creates curtailments in BE too: curtailments are being shared.

Before considering the right-most figure, first consider Figure 4. Like Figure 3 it illustrates the curtailments, but now rather than expressing them in MW they are expressed in percentage of the local PTO volume. Again the left-most figure with the linear penalty function illustrates that curtailments are reduced. The middle figure for the quadratic penalty is more interesting. It still illustrates that curtailments start being shared: curtailment appears for BE too. However the relative curtailments seem disproportionate: French curtailments do not exceed 35%, whereas they can trigger BE curtailments in excess of 50% of the BE PTOs. This actually illustrates the reasons for solution 3 described in section 3.3: penalizing the curtailment **ratios**. Hence the solution is dubbed *ratio* in this text.

Now we can come back to the right-most illustration of Figure 3: when curtailment ratios are minimized the effect on absolute curtailments for France are more modest, since France has a larger market than Belgium. Would the curtailment have originated in Belgium we would have expected absolute BE curtailments to increase.

Finally the right-most illustration of Figure 4 illustrates how this patch manages to reduce the French curtailment ratios (compared to doing nothing), allowing curtailments to be shared with Belgium, while maintaining a better balance in the respective curtailment ratios.

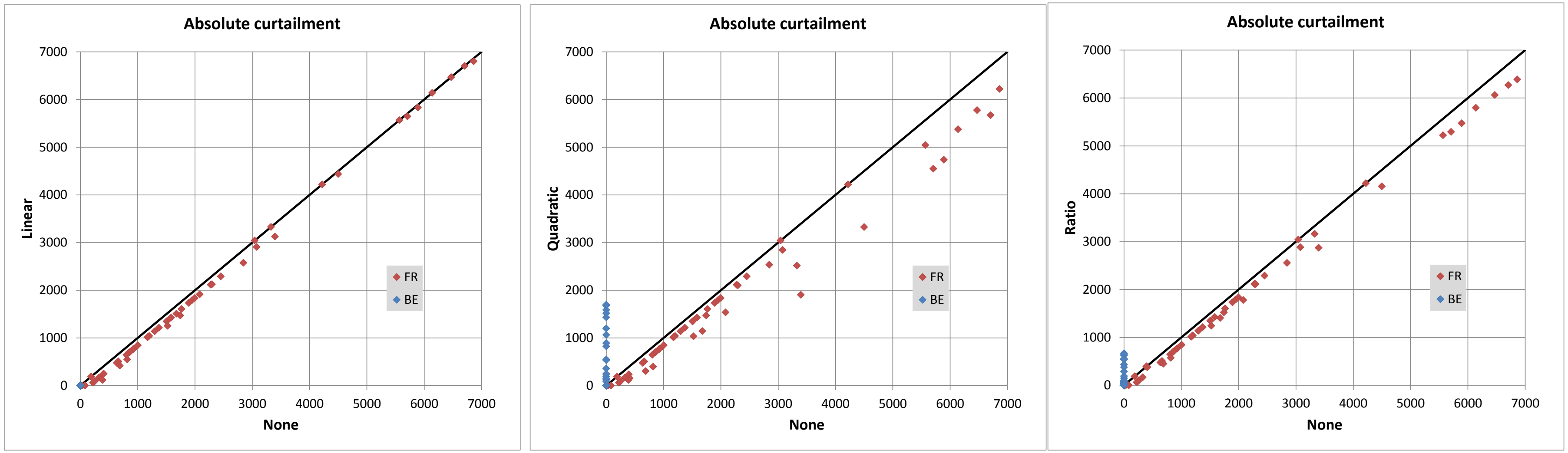


Figure 3 Curtailment (in MW) of the BE and FR markets for the FBI simulations. The cases considered are those for which FR initially (i.e. if no patch would be applied) cleared at 3000€/MWh. The initial curtailments are reflected on the horizontal axis. The vertical axes represent the **absolute** curtailment after application of the patch

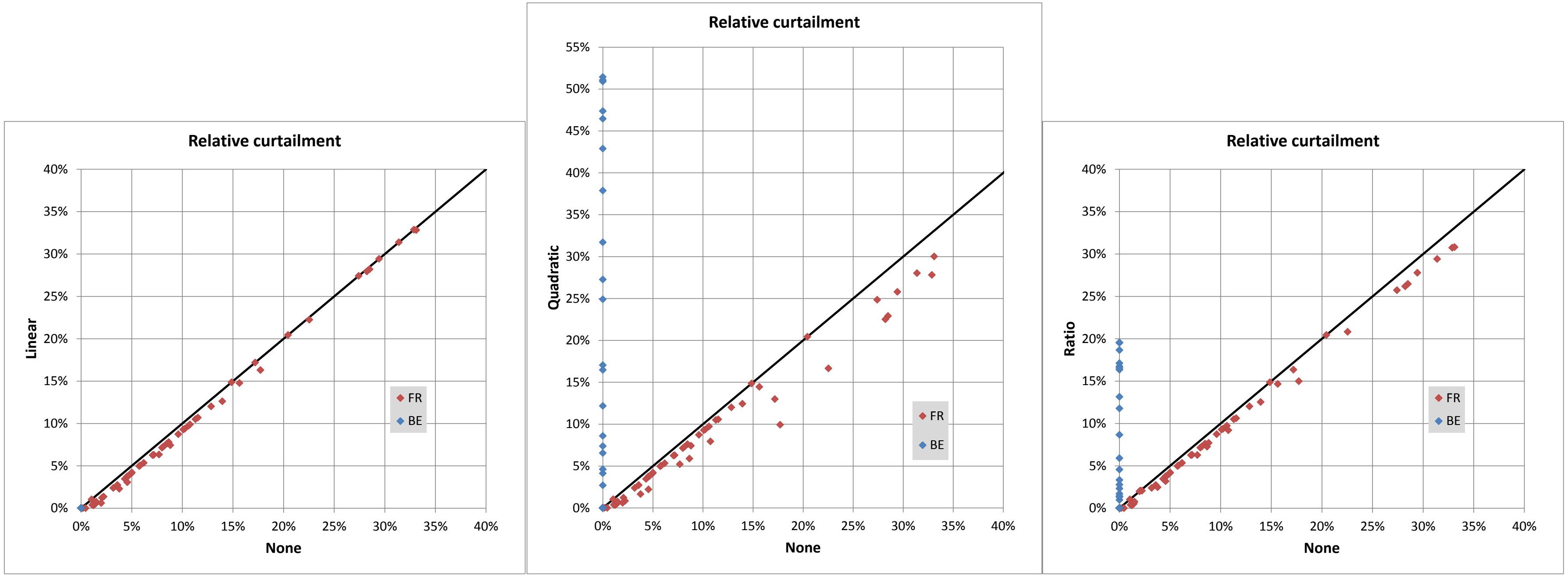


Figure 4 Curtailment of the BE and FR markets for the FBI simulations. The cases considered are those for which FR initially (i.e. if no patch would be applied) cleared at 3000€/MWh. The initial curtailments are reflected on the horizontal axis. The vertical axes represent the **relative** curtailment after application of the patch

4.3.2 Double curtailment cases

This section explores those simulation results for which two markets cleared at 3000€/MWh, whereas all other hubs cleared at lower prices. Here we should find the differences between the linear, quadratic and ratio approaches.

Especially for the case where we added 1000MW load to BE demand, on top of the 1654MW we added to reflect high load, we observed frequently situation for which both BE and FR were in curtailment. We consider the results of the "intuitive" FB results that resulted in 3000€/MWh prices for both BE and FR. We again compare the different algorithmic options (None, Linear, Quadratic and ratio).

Consider Figure 5 where BE and FR curtailments for the different options are presented. Since the focus is on the cases where there is a double curtailment, we expect little difference between the NONE and LINEAR cases, since both allow for "flow factor competition" between PTOs. This is confirmed by the scatter diagram. Furthermore the quadratic approach results in curtailments that move closer to the 45 degree line, i.e. the line where BE and FR curtailments become equal. Still with the quadratic patch activated the FR curtailments exceed the BE ones.

However bear in mind that the FR market is larger than the BE one, so in absolute terms it is only expected to find larger FR curtailments. Instead we consider the same data, but now focussing on the relative curtailments in BE and FR, i.e. the percentage of the price taking orders that has not been matched. This is illustrated in Figure 6. Here some interesting patterns emerge:

1. For a number of hours BE and FR curtailments are equal. This corresponds to periods where the intuitive patch manages to equalize the BE and FR prices (to 3000€/MWh). The intuitive patch discards some relieving effects that do happen, effectively leaving some capacity unused. This unused capacity leaves some freedom to shift energy from one curtailed market to another. The same post-processing step that is used to equalize curtailment ratios under ATC, now uses this freedom to equalize the BE and FR ratios;
2. For some hours the results for the quadratic case are identical to the one from both the NONE and LINEAR results. This can be the case if the LOCAL MATCHING constraint is active: the constraint that prevents local hourly sale to be exported, before it is used to address the local curtailment situation. In this case the curtailments remain fixed;
3. For the remaining hours the curtailment **ratios** deteriorate for BE.

Especially this last point is a shortcoming of the initial prototype that was created to test the viability of the approach. The underlying source is that this prototype penalizes the square of the absolute curtailments. This is different from the approach of the ATC curtailment rules (cf. section 6), where as a post processing the square of the curtailment ratios (weighed with the PTO volumes) is minimized. In the ATC approach the ratios are equalized. By adopting a similar approach for the penalty function it should be possible to recreate this behaviour with the adequacy mitigation.

This finding triggered the development of solution 3 (or *ratio* approach), which penalizes curtailment ratios. Finally Figure 7 illustrates both the absolute and relative curtailments that result under this *ratio* approach compared to the *quadratic* approach. As expected the absolute French curtailments increase (due to the larger market size), but this is a consequence of the curtailment ratios moving closer. This last point is best illustrated in the curtailment ratios scatter of Figure 7: the *ratio* patch clearly results in much more comparable curtailment ratios between the two curtailed markets. Although we can observe that results are much closer to the 45 degree line, not all points coincide with it. Note that this is consistent with expectations: if curtailment ratios aren't identical, this results in a higher penalty function. However due to more or less favourable flow factors this might still be beneficial. The direction in which the ratio penalty function works is to equalize ratios more.

Also consider section 7, where a simplified model of the ratio solution is presented. Here it is demonstrated that an expression for the resulting curtailment ratios can be derived and depends both on the volume of the PTOs as well as the values of the PTDFs.

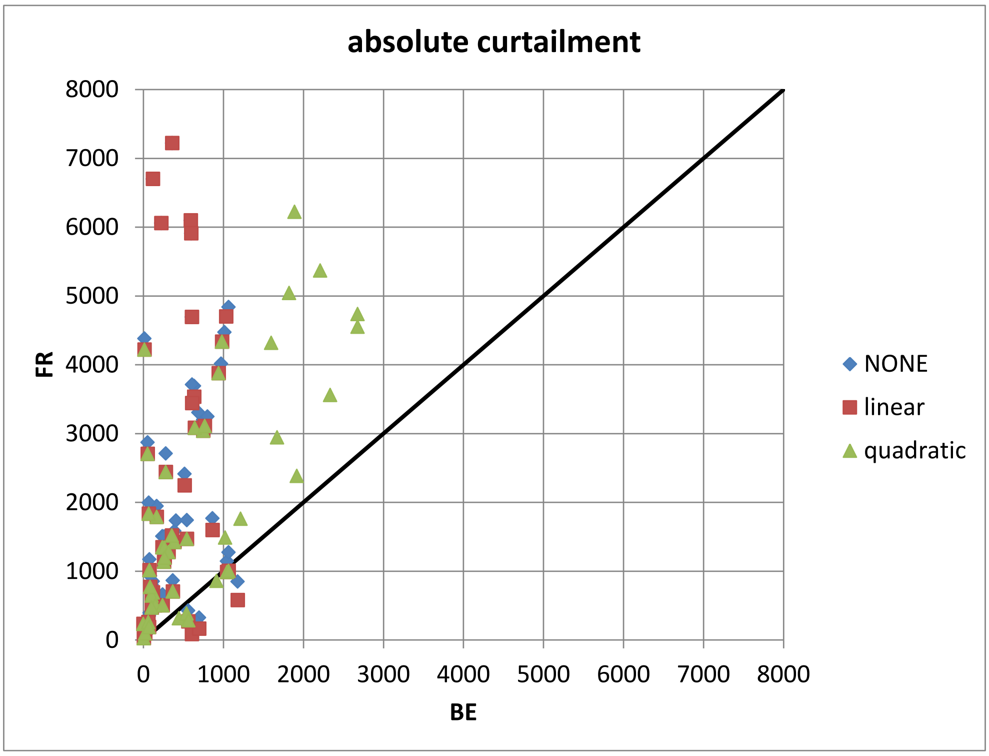


Figure 5 Order curtailments of BE and FR for FBI simulations where 1000MW extra demand has been added in BE on top of the already stressed assumptions for both BE and FR.

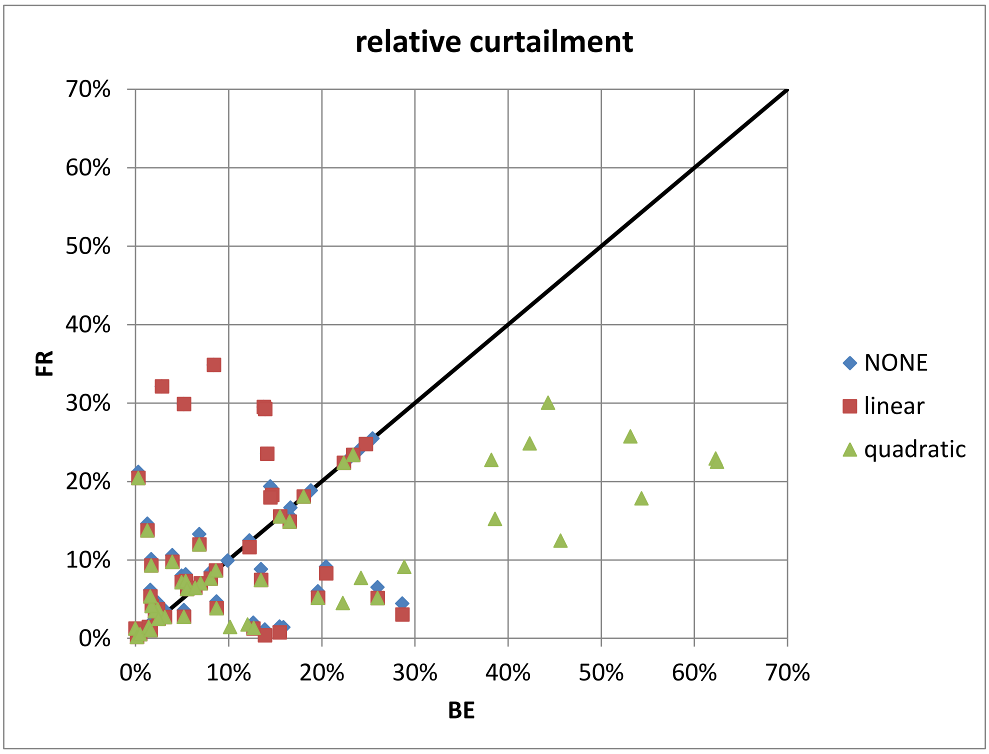


Figure 6 Relative order curtailments of BE and FR for FBI simulations where 1000MW extra demand has been added in BE on top of the already stressed assumptions for both BE and FR.

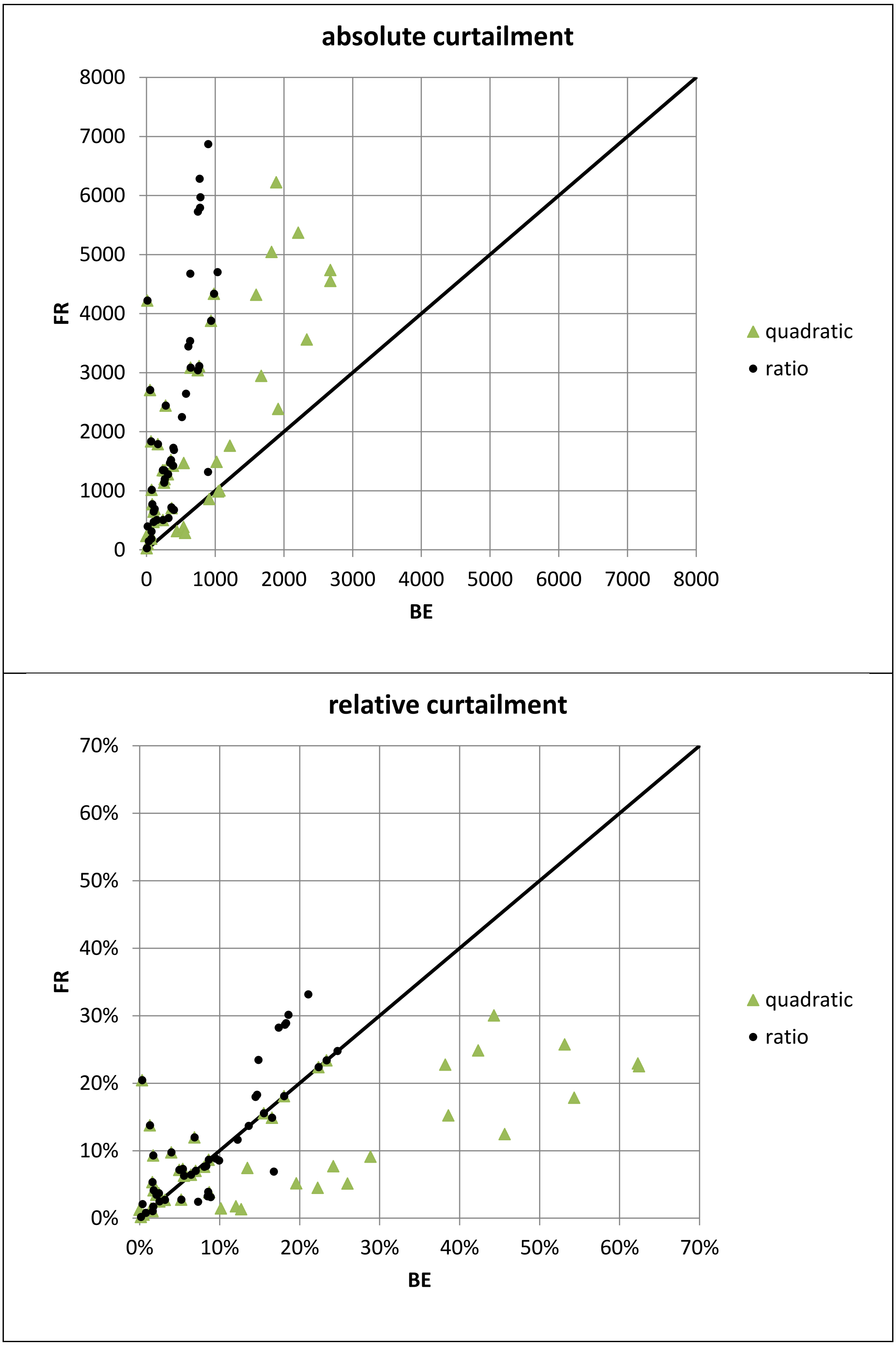


Figure 7 Curtailments of BE and FR for FBI simulations where 1000MW extra demand has been added in BE on top of the already stressed assumptions for both BE and FR. The focus is on results using the *quadratic*, respectively *ratio* patch.

4.3.3 Impact on overall curtailments

It is important to note that any sharing of the curtailments results in an increase of the overall curtailment: if "flow factor competition" between price taking orders prefers import in one market over import in another, this implies that the first market can import more. If we now also allow this second market to import some energy, by definition the combined import decreases and therefore the overall curtailment increases. We expect this effect to show up when comparing the absolute curtailments between the linear and quadratic approaches.

Figure 8 illustrates the BE+FR total curtailment comparing the linear approach with the quadratic approach and the ratio approach. We distinguish two types of solution:

1. Some solutions end up on the diagonal: both approaches result in identical levels of curtailment. This relates to the "LOCAL MATCHING" constraint (cf. section 6): a constraint that prohibits locally available hourly supply to be exported, whereas simultaneously local price taking demand needs to be curtailed. In case this constraint is hit, the solution is fully determined and consequently there is no difference between the quadratic and linear solutions.
2. For all solutions not on the diagonal we observe that the overall curtailment increases when switching from the linear penalty to the quadratic penalty. This confirms our theoretical expectation. This holds true for both the quadratic and ratio approaches.

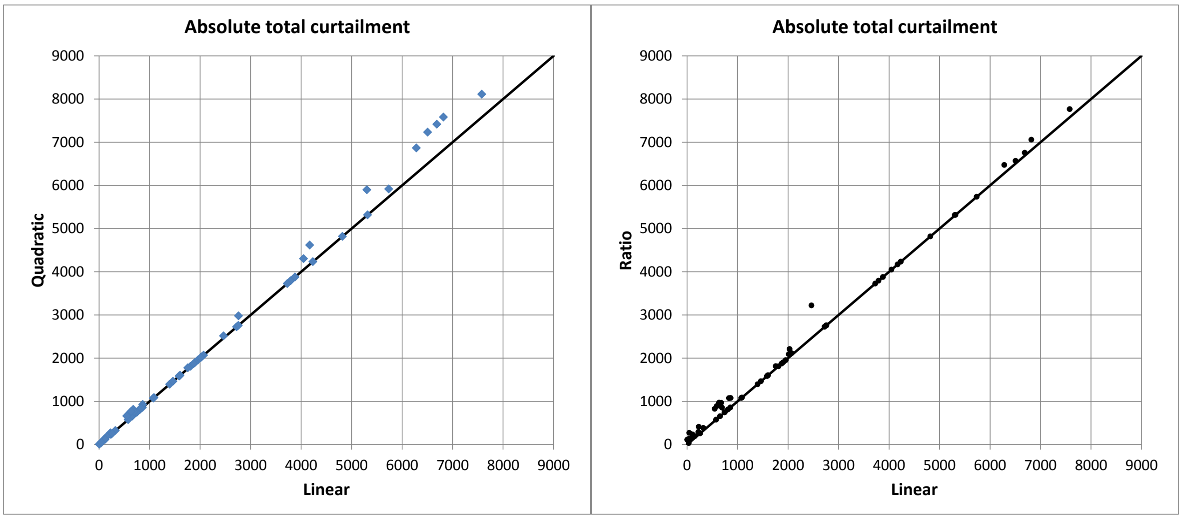


Figure 8 Illustration of the total curtailment (BE+FR) between the linear and quadratic penalty for FBI simulations where 1000MW extra demand has been added on top of the already stressed assumptions for both BE and FR.

4.4 Interaction with local matching

In the preceding sections as well as in the description of the current ATC curtailment rules (cf. section 6) some mention was made of the "local matching" constraint. This section is dedicated to better explain what this constraint is and what its place is in relation to the curtailment sharing functionality and the way in which it interacts with the adequacy mitigation.

This section introduces what is meant with how Euphemia treats order curtailment situations in a three tier approach, and develops examples of the interplay with different configurations:

- Isolated markets;
- ATC coupled markets in the absence of harmonized price boundaries;
- FB coupled markets;
- FB coupled markets where an adequacy mitigation exists;

Euphemia curtailment

We consider order curtailment to be the situation where orders submitted at an extreme price (e.g. sell orders at -500€/MWh or buy orders at +3000€/MWh) cannot be fully matched. We further precise the notion of price taking orders as those orders that do not add any additional constraints (i.e. block orders are never considered as price taking orders since they add fill-or-kill constraints). In the context of the adequacy mitigation we will only focus on the curtailment situations at 3000€/MWh.

With regards to curtailment the following three tier process is implemented in the matching algorithm before and after the welfare maximisation:

Step 1: Generation of the local Matching constraint

Step 2: Welfare maximisation[3] (subject to the local matching constraint)

Step 3: Lifting volume indeterminacies and equal sharing of curtailment

- Under ATC the local matching constraint is disabled to allow equal curtailment sharing with respect of the ATCs
- Under FB the local matching constraint is disabled to allow equal curtailment sharing with respect of the FB constraints.

Local matching constraint (demand side)

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[3] With the "patch", Step 2 is the welfare maximization and PTO curtailment ratio minimisation

The local matching constraint considers to what extent the local buy PTOs can be matched by the locally available hourly supply orders. This is illustrated in Figure 9: the left hand side illustrates the situation where the total hourly supply volume exceeds the demand PTOs, whereas the right hand side illustrates the case where the hourly supply volume can only cover part of the demand PTOs. The local matching constraint forces solutions to always cover at least that part of the PTO volume that can be locally met, i.e. the smaller of the PTO volume and the total hourly supply. This constraint is generated for each area individually.

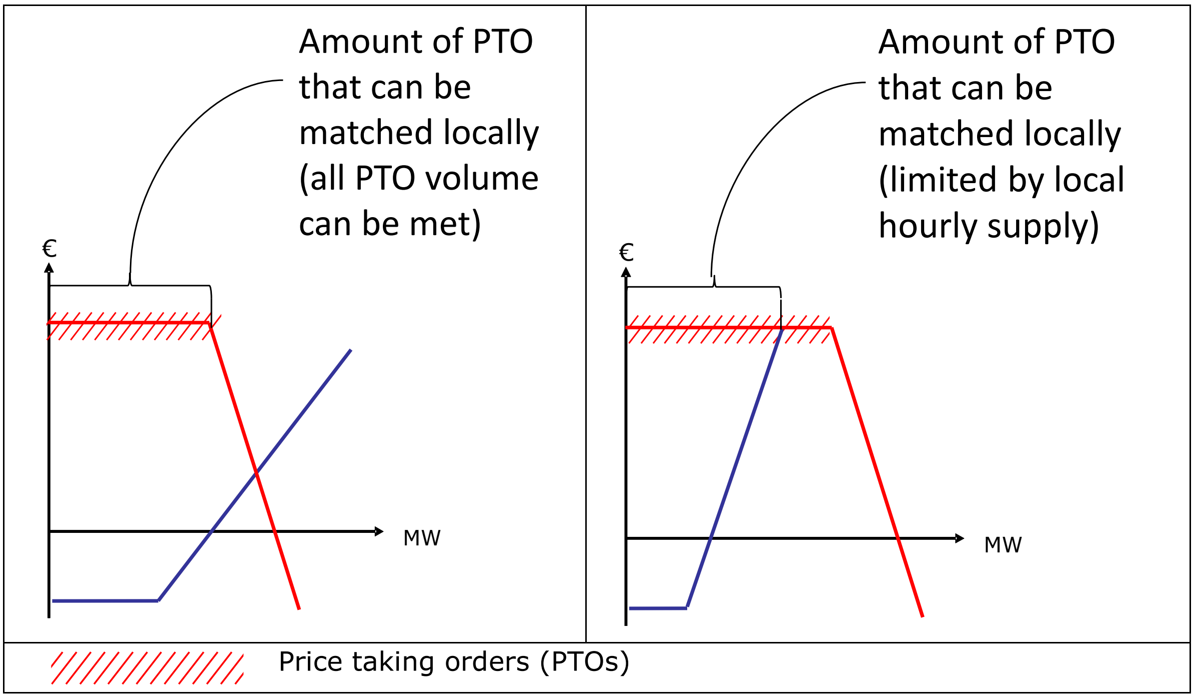


Figure 9 Illustration local matching constraint

The reason to impose the local matching constraint is to:

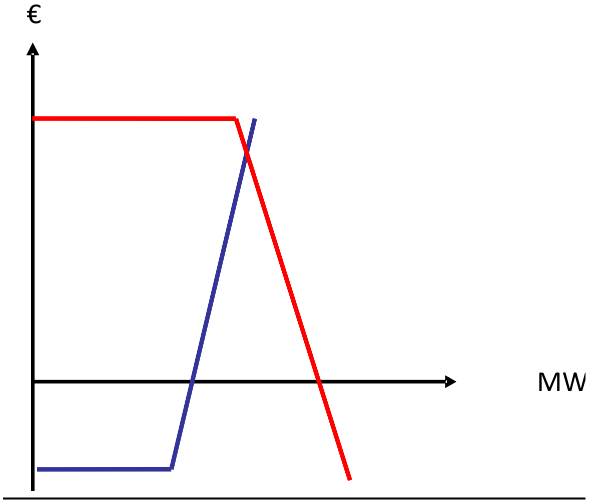
- Favour hourly price taking order (said PTO) execution over block order execution,
- Prevent orders of areas with the same or wider price bounds from outbidding the hourly price taking orders of areas with the same or tighter price bounds.

Examples

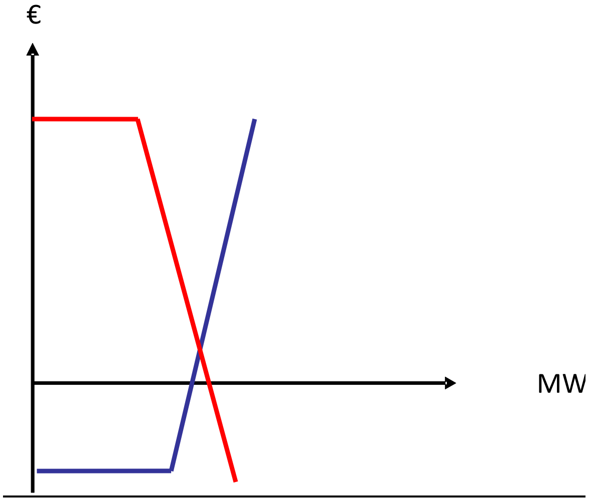
To get a better grasp of how the local matching constraint fits in the Euphemia calculations, the next sections consider a series of examples.

4.4.1 Isolated markets

This example considers a single market and two hours. The hourly orders are illustrated in the curves below:



Hour 1

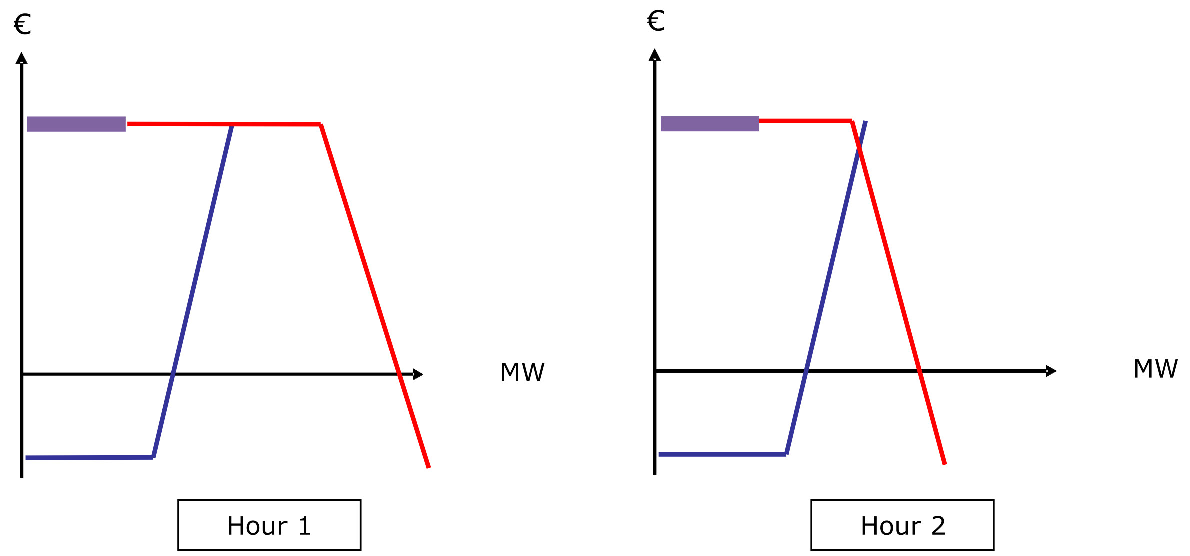


Hour 2

Apart from these curves the example assumes:
100MWh demand block order that spans hours 1 and 2 and is submitted at 3000€/MWh.

STEP 1: local matching constraint
Since both hours have ample hourly supply, the local matching constraints dictates that all PTO demand should be accepted.

STEP 2: welfare maximisation



In this simple example the welfare maximisation comes down to trying to accept the one block order (i.e. the 100MW block volume enters both hours). If we try to accept the block (see above illustration), consequently the PTO in hour 1 is being curtailed. Since this violates the local matching constraint that was generated in the previous step (recall we said no PTO may be curtailed), the block order is rejected.

STEP 3: Lifting of volume indeterminacies and equal sharing of curtailment

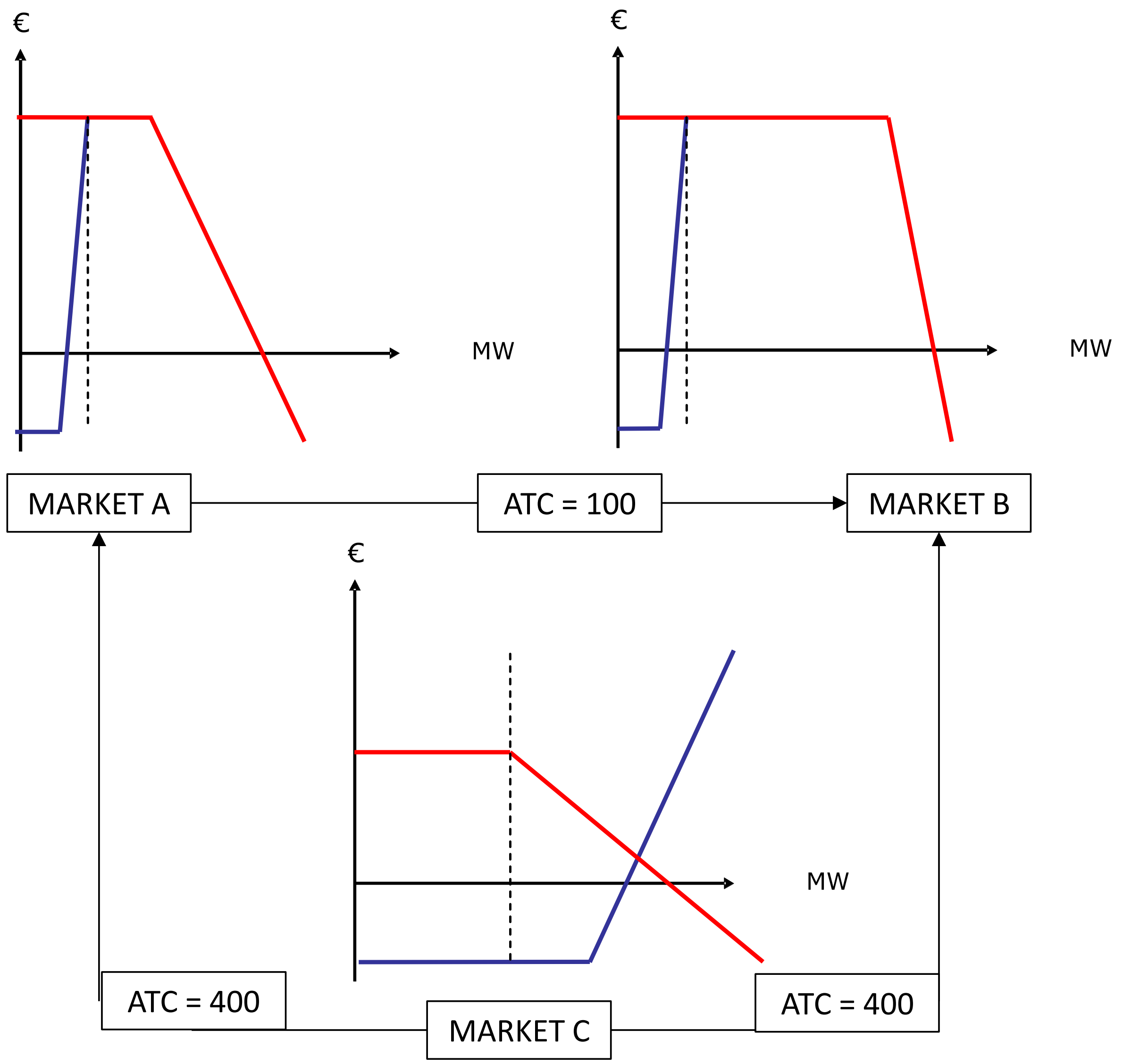
This step is irrelevant in the case of an isolated market.

Conclusion

The isolated example illustrates how the local matching constraint secure the PX market rule to favour hourly PTO execution over block order execution in curtailment situation.

4.4.2 ATC coupled markets in the absence of harmonized price boundaries

This example considers a one hour problem, using a three market topology. Markets A and B have a higher maximum price than market C. No block orders are considered, only hourly orders. The example is illustrated in the figure below:



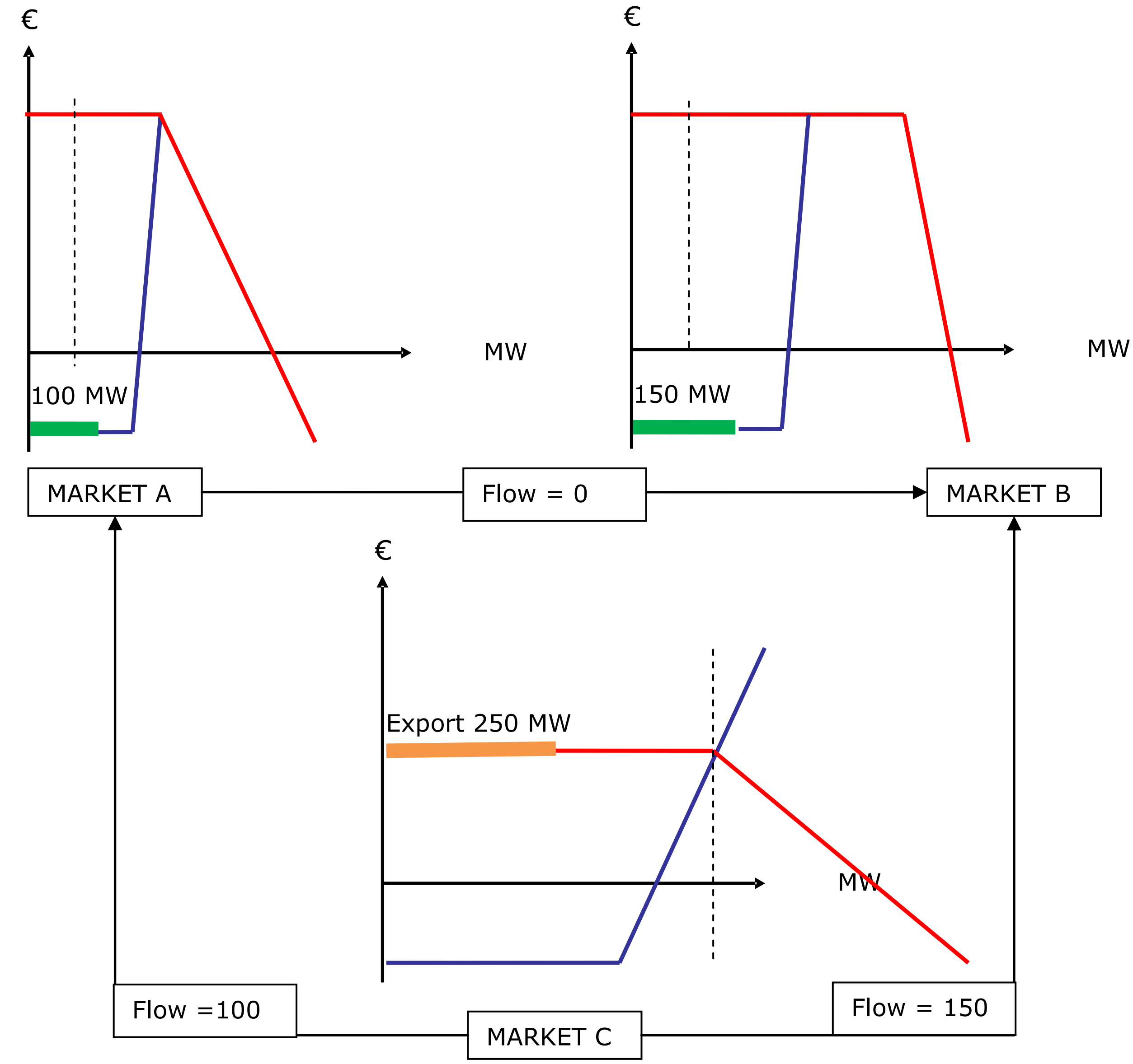
<u>STEP 1: local matching constraint</u>

In the illustration above, for each market the local matching constraint (minimum quantity of PTO that needs to be accepted) is illustrated by the dashed lines;

<u>STEP 2: welfare maximisation</u>

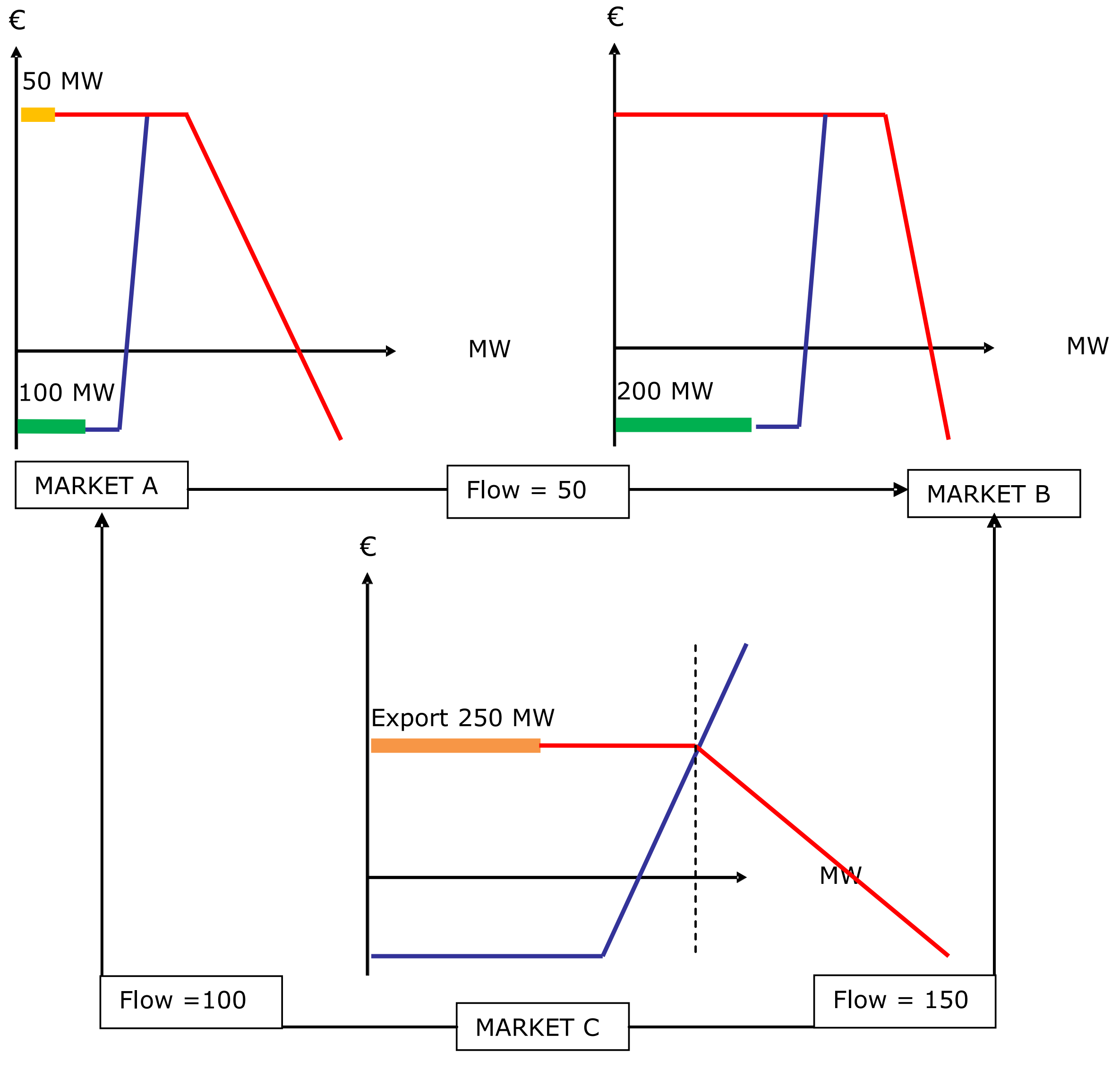
The welfare maximising exchanges are illustrated below. Note that from a pure DAMW perspective it would be favourable to let market C export more (curtailing the PTOs of market C that are priced lower than the PTOs of either market A or B results in an increase in welfare). However if we consider orders submitted at maximum price to be something akin to lost load cases, the welfare definition breaks down, and we want to prevent the curtailments. This is precisely what the local matching constraint forbids:

market C is only allowed to export by curtailing its demand with price indication. Price taking demand is not allowed to be curtailed.



STEP 3: Lifting volume indeterminacies and equal sharing of curtailment

Finally for those markets that have the widest price bound (markets A and B in the example) the curtailments are shared (subject to the network constraints). The solution for market C is fixed to the one from the previous step (i.e. it will export 250MW). To this end the local matching constraints is relaxed and 50MW of the imports from market A are funnelled to market B, resulting in equal curtailment ratios between markets A and B, lifting the bias in curtailment from the previous step.



Conclusion

Under ATC coupling the local matching constraint prohibits markets which have tighter price bounds from importing curtailments. At the same time the curtailment sharing step allows those markets with wider bounds to share curtailments (to the extent network constraints allow this).

4.4.3 FB Coupled markets – no patch

For the FB example we consider again a three market example, characterized by the curves that are illustrated in the figure below this text:

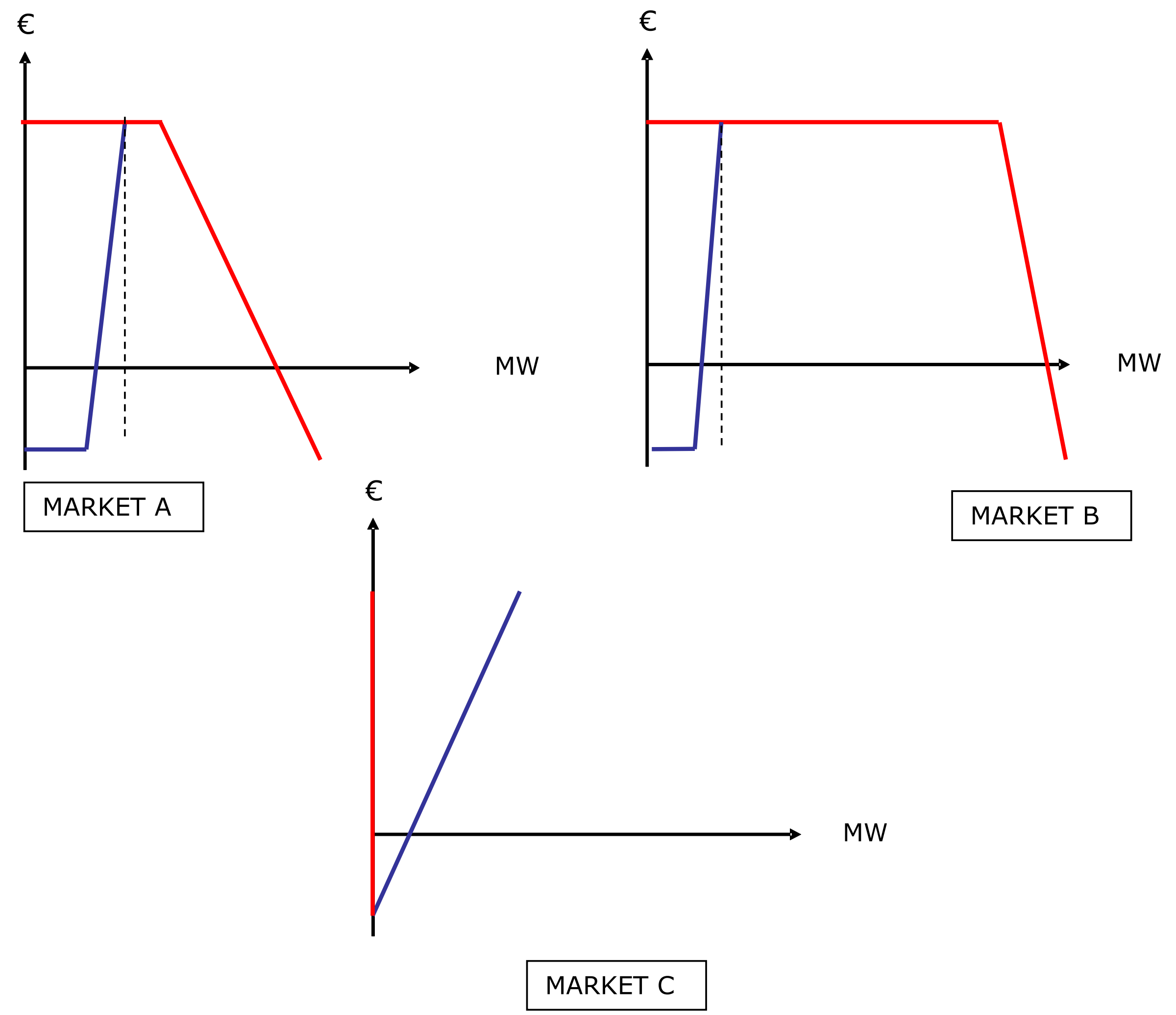
- Market A has 50MW initial curtailment (= 25% of PTO);
- Market B has 400MW initial curtailment (= 80% of PTO);
- Market C: no demand, just 250MW of sell volume;

The zone to zone FB constraint is:

$PTDF_{C \rightarrow A} = 0.2$;

$PTDF_{C \rightarrow B} = 0.5$;

RAM = 50MW;



STEP 1: local matching constraint

The local matching constraints as indicated by the dotted line in the graphs are:

A: $PTO_A \geq 150$;

B: $PTO_B \geq 100$;

STEP 2: welfare maximisation

Since market C clears well below the 3000€/MWh of markets A and B, it will become the exporting hub. Due to the fact that exports from C to A have a more favourable flow factor, this will be the preferred direction: market A has the same high price as market B, but it can import more. So market A will import 50/0.2 = 250MW.

Note that from a welfare maximisation point of view it would even be beneficial to let market B **export** energy: an export from B to A costs:

$B \rightarrow C + C \rightarrow A = -0.5 + 0.2 = -0.3$

I.e. for each MW from B to A the congestion is relieved by 0.3MW, whereas there is no loss of welfare (curtailment in B is aggravated, but in A it is reduced by the same amount). The 0.3MW of freed capacity can subsequently be used to exchange 1.5MW between market C and B, which will contribute positively to welfare.

However in order to force market B to export we need to increase its curtailment, which is precisely what is forbidden by the local matching constraint. Therefore the welfare maximising solution is indeed to just send 250MW from C to A, and deny market B any import.

STEP 3: Lifting volume indeterminacies and equal sharing of curtailment

Finally the local matching constraint is dropped and we will try to share the curtailments. The only markets in curtailment are markets A and B, whereas the net position of market C is fixed. The FB constraint was already fully saturated. To more equally share the curtailments we wish the export of some energy from market A to market B. But a 1MW $A \rightarrow B$ exchange uses 0.3MW of the already tight constraint. Hence the curtailment sharing step fails to bring any additional value under this FB scenario.

Conclusion

The local matching constraint under a FB model, without adequacy mitigation will prevent markets that are in curtailment and have unfavourable flow factors, to be forced to export and hence increase their curtailment.

4.4.4 FB coupled market - patched

We consider the same example as in the previous case, but now consider the FB model where the patch has been implemented. From our earlier analysis we know that for the double curtailment case there will not be any difference using the unpatched algorithm or the linear penalty function. Therefore rather than the linear approach, we will consider the ratio approach from solution 3.

4.4.4.1 Base case – local matching constraint not active

Recall that the change to the objective function (welfare maximisation) is to add a penalty term:

Quadratic objective – solution 3
max welfare – M * PTO volume * (rejected PTO ratio)^2

Due to the big M, the penalty will dwarf the welfare, so it is safe to discard the precise prices of the orders in market C (which does not face a curtailment situation).

Rather than using the zone-to-zone PTDFs, we will use a direct PTDF. We choose zone C as a slack "node":

$PTDF_C$ = 0; $PTDF_A$ = -0.2; $PTDF_B$ = -0.5;

We can now derive the mathematical problem that the ratio approach will create:

$\min 200 \cdot (1 - x_A)^2 + 500 \cdot (1 - x_B)^2$: with 200 PTO in A; 500 PTO in B

s.t.

$nex_A = 150 - 200 \cdot x_A$ (1): with 150 the local sell volume, 200 the PTO volume

$nex_B = 100 - 500 \cdot x_B$ (2): with 100 the local sell volume, 500 the PTO volume

$nex_C = -nex_A - nex_B$ (3): the balance constraint

$-0.2 \cdot nex_A - 0.5 \cdot nex_B \leq 50$ (4): the (active) CB

$200 \cdot x_A \geq 150$ (5): local matching constraint market A

$500 \cdot x_B \geq 100$ (6): local matching constraint market B

Where x_A and x_B are the fraction of PTOs matched. Solving for this problem yields as a solution:

$x_A = 0.76$; $nex_A = -1.9$;

$x_B = 0.40$; $nex_B = -99.2$;

$nex_C = 101.1$;

Both markets import energy, i.e. the local matching constraints were not hit. So in this case the local matching constraint had no impact. To make for more interesting cases, the next section explores a case where the local matching constraint is hit.

4.4.4.2 Case 2 – Local matching constraint active

To illustrate the impact the local matching constraint could have, we now assume that the flow factors for market A and B are reversed (i.e $PTDF_A = -0.5$ and $PTDF_B = -0.2$);

Solving again now yields:

$x_A = 0.75$; $nex_A = 0$;

$x_B = 0.70$; $nex_B = -250$;

$nex_C = 250$;

Clearly in this case the local matching constraint was active: it prevented market A from exporting, thereby increasing the curtailment in market B.

4.4.4.3 Case 3 – Local matching deactivated on case 2 example

Finally we reconsider the example of case 2, by solving again the same model, but now deactivating the local matching constraints. Results are:

$x_A = 0.61$; $nex_A = 28.6$;

$x_B = 0.84$; $nex_B = -321.4$;

$nex_C = 292.8$;

Without the local matching constraint, market A would be forced to export some of its energy. For this example the disparity in curtailment ratios actually increased when removing the local matching constraint. Of course other examples exist illustrating precisely the opposite effect.

Conclusion

The interaction of the local matching constraint with the adequacy patch can be quite complex, even for seemingly trivial examples. Depending on the precise flow factors of the active CBs, and the amount of the respective curtailments, a local matching constraint may or may not become active. Since the local matching constraint is active during the welfare maximisation step occasionally it will limit curtailment sharing, i.e. for these cases the mitigation will be less active.

4.4.5 Final consideration – block orders

Finally we point out that the local matching imposes the PTO volume to be accepted at least to the extent the market can meet this demand through its locally available **hourly** supply. Note that block supply volume is not included in this constraint. This would not be possible, because - due to the fill-or-kill constraint - it is hard to anticipate which block will, and which block will not be accepted. However it seems reasonable to assume that in case of curtailment situations and clearing prices hitting 3000€/MWh, some sell blocks will be accepted. Since this will be volume above the hourly supply, this volume can be exported without violating local matching constraints.

5 Mitigation proposal

To address the issues identified by NRAs related to “flow factor competition” where it concerns price taking orders, the project suggests implementing the solution described in 3.3. Section 3.2 demonstrates this mitigation be effective against price taking orders being outcompeted by non-price taking orders in adjacent markets. The synthetical example developed in section 7.1 illustrates some bias against smaller markets under the solution described in section 3.2, which is lifted with solution from section 3.3. It is the understanding of the project that the associated loss in welfare and increase of overall curtailment is an acceptable price for this mitigation measure.

Unlike the first alternative that was explored (cf. section 3.1) this solution does manage to address the issue of curtailment sharing: solutions to this model still depend on the flow factors of the active critical branches (which could be favourable for either market, depending on its location), but the bias favouring larger markets is addressed.

5.1 Caveats

It should be noted that any changes to the Euphemia algorithm will be implemented by the PCR project, and the PCR project reminds that:

- The proposal requires changes to Euphemia that will need to be properly assessed and approved under the PCR/MRC Change Control process;
- In particular, the following is yet to be assessed:
    1. The impact on other markets and Euphemia performance; and
    2. The ability to guarantee implementation ahead of winter 2015/16;
- The PCR parties will make reasonable efforts to further progress these issues in parallel with the NRA consultation process.

CWE project partners are confident that a solution will be available for winter 2015/16 and committed to secure it.

6 Annex – current curtailment rule

The curtailment situation pre-dates FB and exists under ATC as well. However unlike FB, there exists no "flow factor competition" under ATC, making it possible to treat a curtailment sharing as a post-processing step in the algorithm: if two or more markets clear at maximum price, and not all PTOs can be filled, there exists some freedom where to allocate the energy. Since all energy was entered at the same 3000€/MWh price, changing the allocation does not affect welfare.

As far as curtailment is concerned we can therefore distinguish two main phases in the algorithm:

1. Find a welfare maximising solution;
2. In case the solution under 1. resulted in curtailment, see if it can be allocated across markets such that the curtailment ratios are as equally as possible[4];

Finally we remind you that in step 1. there also is the LOCAL_MATCHING constraint, which prohibits locally available hourly supply to be exported if simultaneously local price taking demand needs to be curtailed. This constraint finds its origin in a configuration without harmonized extreme prices. The LOCAL_MATCHING constraint prevents areas with wider bounds from outbidding the price takers of areas with more narrow bounds. For the areas with the widest bounds the constraint is relaxed in the second problem, to allow for curtailment sharing.

With today's harmonized price bounds, the constraint is somewhat redundant, since it will be relaxed in step 2.

I.e. the curtailment sharing under ATC tries to equalize the curtailment ratios. More details can also be found in section 6.5 of the public Euphemia description:

[4] In fact a subsequent problem is solved, under all physical constraints, fixing the welfare and minimizing $\sum_z Q_z^{PTO} \cdot \left(1 - x_z^{PTO}\right)^2$, i.e. minimize the squared non-acceptance of PTOs, weighed with the PTO volume. It follows that in case there are no binding constraints, this will result in equal curtailment ratios in the affected markets.

http://www.apxgroup.com/wp-content/uploads/Euphemia-public-description-Nov-20131.pdf

7 Annex - Simplified model

Solutions 2 and 3 both model a quadratic penalty function, but differ in one detail: solution 2 penalized the square of the curtailment, whereas solution 3 penalizes the square of the curtailment ratio weighed with the price taking order. To allow for a comparison, in this section we develop the optimality conditions for a strongly simplified example.

Both solutions weigh the penalty with a factor big M. If we assume the M to be sufficiently large, it will dominate the welfare term. If we further assume two or more markets are in curtailment, but only 1 market exports, then we can equivalently write the problem as one that minimizes the curtailment penalty. Note that the assumption of a single export market is necessary: if two or more markets export, there will be flow factor competition deciding which market export what. In our simple model we assume only 1 exporting market and we only consider critical branches that become active and write:

Solution 2	Solution 3
$\min \sum_z \left(PTO_z \cdot (1 - x_z)\right)^2$ s.t.	$\min \sum_z PTO_z \cdot (1 - x_z)^2$ s.t.
$\sum_z \left(PTDF_i^{cb} - PTDF_z^{cb}\right) \cdot PTO_z \cdot x_z = RAM$	$\sum_z \left(PTDF_i^{cb} - PTDF_z^{cb}\right) \cdot PTO_z \cdot x_z = RAM$

Where:

i: index for exporting market;

z: index for importing markets;

PTO_z: volume of the price taking orders in market z;

x_z: acceptance ratio of price taking order in market z;

$PTDF_k^{cb}$: flow factor for market k for critical branch cb;

RAM: remaining available margins.

To illustrate flow factor competition between PTOs, it suffices if we only consider two curtailed markets, and just 1 critical branch. We hence further simplify our model:

The two markets in curtailment are A and B, the exporting market is C. We introduce as a shorthand $PTDF_{ij} = PTDF_i - PTDF_j$.

Solution 2	Solution 3
$\min\left(PTO_A \cdot (1-x_A)\right)^2 + \left(PTO_B \cdot (1-x_B)\right)^2$ s.t.	$\min PTO_A \cdot (1-x_A)^2 + PTO_B \cdot (1-x_B)^2$ s.t.
$PTDF_{CA} \cdot PTO_A \cdot x_A + PTDF_{CB} \cdot PTO_B \cdot x_B = RAM$ (1)	$PTDF_{CA} \cdot PTO_A \cdot x_A + PTDF_{CB} \cdot PTO_B \cdot x_B = RAM$ (1)

From (1) we isolate x_B:

$$x_B = \frac{RAM - PTDF_{CA} \cdot PTO_A \cdot x_A}{PTDF_{CB} \cdot PTO_B}$$

And are left with:

Solution 2	Solution 3
$\min\ f(x_A) = \left(PTO_A \cdot (1-x_A)\right)^2 + \left(PTO_B \cdot \left(1 - \frac{RAM - PTDF_{CA} \cdot PTO_A \cdot x_A}{PTDF_{CB} \cdot PTO_B}\right)\right)^2$	$\min\ f(x_A) = PTO_A \cdot (1-x_A)^2 + PTO_B \cdot \left(1 - \frac{RAM - PTDF_{CA} \cdot PTO_A \cdot x_A}{PTDF_{CB} \cdot PTO_B}\right)^2$

For a minimum of $f(x_A)$ we need:

Solution 2:

$$\frac{\partial f}{\partial x_A} = 0:$$

$$\frac{2 \cdot PTO_A \cdot PTO_B \cdot PTDF_{CA} \cdot \left(1 - \frac{RAM - PTO_A \cdot PTDF_{CA} \cdot x_A}{PTO_B \cdot PTDF_{CB}}\right)}{PTDF_{CB}} - 2 \cdot (PTO_A)^2 \cdot (1-x_A) = 0$$

$$\Rightarrow x_A = \frac{RAM \cdot PTDF_{CA} + PTO_A \cdot (PTDF_{CB})^2 - PTO_B \cdot PTDF_{CA} \cdot PTDF_{CB}}{PTO_A \cdot \left((PTDF_{CA})^2 + (PTDF_{CB})^2\right)}$$

$$\Rightarrow x_B = \frac{RAM \cdot PTDF_{CB} + PTO_B \cdot (PTDF_{CA})^2 - PTO_A \cdot PTDF_{CA} \cdot PTDF_{CB}}{PTO_B \cdot \left((PTDF_{CA})^2 + (PTDF_{CB})^2\right)}$$

Solution 3:

$$\frac{\partial f}{\partial x_A} = 0:$$

$$\frac{2 \cdot PTO_A \cdot PTDF_{CA} \cdot \left(1 - \frac{RAM - PTO_A \cdot PTDF_{CA} \cdot x_A}{PTDF_{CB} \cdot PTO_B}\right)}{PTDF_{CB}} - 2 \cdot PTO_A \cdot (1 - x_A) = 0$$

$$\Rightarrow x_A = \frac{RAM \cdot PTDF_{CA} + PTO_B \cdot (PTDF_{CB})^2 - PTO_B \cdot PTDF_{CA} \cdot PTDF_{CB}}{PTO_A \cdot (PTDF_{CA})^2 + PTO_B \cdot (PTDF_{CB})^2}$$

$$\Rightarrow x_B = \frac{RAM \cdot PTDF_{CB} + PTO_A \cdot (PTDF_{CA})^2 - PTO_A \cdot PTDF_{CA} \cdot PTDF_{CB}}{PTO_A \cdot (PTDF_{CA})^2 + PTO_B \cdot (PTDF_{CB})^2}$$

For this strongly simplified example we already find that for both quadratic models the relationship between flow factors, volumes of price taking orders, remaining available margins and resulting acceptance ratios (and consequently rejection ratios) of price taking orders is not a trivial one.

7.1 Qualitative evaluation solution 3

Using the model established in the previous section, it is possible to have a more qualitative assessment for solution 3 (ratio): the alternative approach to a quadratic penalty. This option still implements a quadratic penalty, but on curtailment ratios (weighed with the respective price taking volume), rather than the absolute curtailments. This section contains a qualitative reasoning about the solutions to expect, and concludes that no a-priori statements can be made whether solution 2 or solution 3 will result in higher overall curtailments.

We consider the simplified example that is developed in annex 7. The example is limited to three markets, one exporting market, and two importing markets in curtailment. We will make some additional assumptions, namely:

- Values for the zone-to-zone flow factors:
  - $PTDF_{C \to A}$ = 0.4;
  - $PTDF_{C \to B}$ = 0.2;
- Values for the price taking orders:
  - We consider market A small: 1500MWh of PTOs;
  - We consider market B large: 4500MWh of PTOs;
- A value of the remaining available margin (arbitrarily set to 1000MW);

Our example is illustrated in Figure 10.

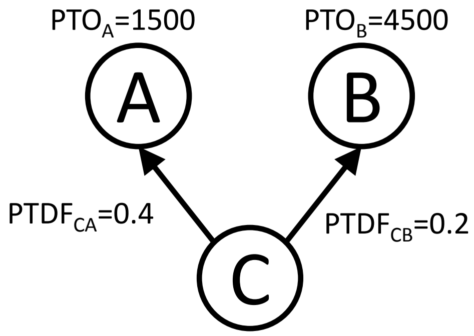


Figure 10 Illustration of the example

Apart from the above example, we create an alternate one, which is obtained by simply switching the flow factors. In the current configuration market B has a more favourable flow factor. For the alternate configuration market A will receive the more favourable flow factor.

Table 1 illustrates the results. The grey boxes illustrate the curtailments both in absolute and relative terms.

If we consider the total curtailments, we find:

Example	Solution 2	Solution 3
1	1500MWh	1785MWh
2	3300MWh	2961MWh

I.e. for example 1 it is solution 2 that results in the lowest overall curtailments, whereas for example 2 it is solution 3 that has lower overall curtailments. Depending on the flow factors and the volume of PTOs in each market, the comparison will vary.

Finally we note that solution 3 indeed better manages the discrepancy in curtailment ratios than solution 2 does. This mainly favours the smaller market: it still receives less energy than the larger market, but its relative weight is now being considered. In case the smaller market has a more favourable flow factor (example 2) it still receives less imports, but due to its favourable flow factor manages to have less (relative) curtailments compared to the larger market. This unlike solution 2, where it was actually penalized for being a small market (under solution 2 market A receives less import when it has a more favourable flow factor).

	Assumptions	Solution 2 minimize absolute curtailments	Solution 3 minimize curtailment ratios

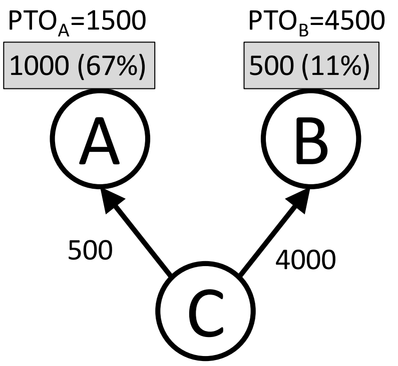
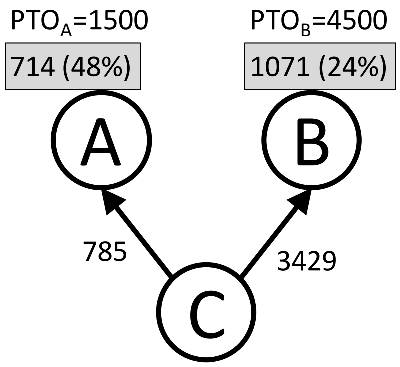
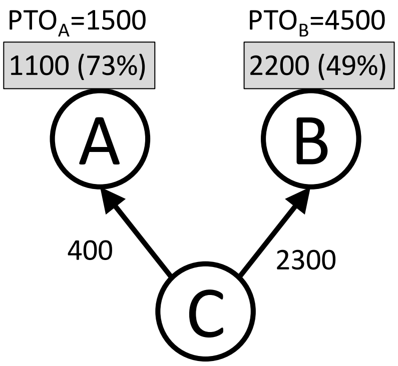
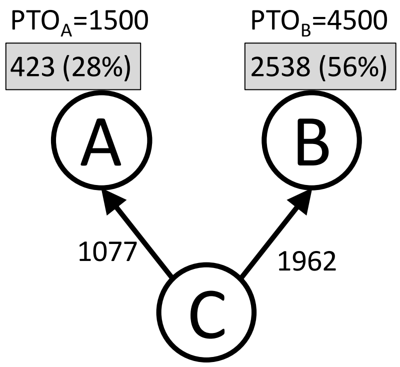
Example 1	$PTDF_{C \to A}=0.4$ $PTDF_{C \to B}=0.2$		
Example 2	$PTDF_{C \to A}=0.2$ $PTDF_{C \to B}=0.4$		

Table 1 Examples of results under solutions 2 and 3 (sharing absolute curtailments vs relative curtailments). The grey boxes denote the curtailments (both absolute and as a percentage of the total curtailments)