Specification for intuitive BEC

1 Introduction

The current approach to compute bilateral exchanges could result in non-intuitive bilateral exchanges, even when FBMC is run in FB "intuitive" mode. Since such results might appear inconsistent with the project's choice to go live in FB "intuitive" mode, it was decided to issue this change, to retrieve BECs that are intuitive too.

2 BEC requirement

Input

Input for the BEC calculation are the 4 CWE net positions (nex_z) and the 4 CWE market clearing prices (mcp_z)

Output

The 8 directional CWE bilateral exchanges

Method

STEP1 – guaranteeing the net positions sum to zero

The balance constraint is not necessarily respected by the input data. This can be restored by applying the method described in section 3.

STEP2 – deriving bilateral exchanges

We define:

$$x^* = \frac{2\,nex_{BE} + nex_{NL} - nex_{FR}}{4}$$

And we derive a lower bound LB and an upper bound UB as follows:

LB = -∞	
UB = ∞	
LB:= max(LB, 0)	If $mcp_{FR} - mcp_{BE} < \varepsilon$
LB:= max(LB,-nex_{BE})	If $mcp_{BE} - mcp_{NL} < \varepsilon$
LB:= max(LB,-nex_{BE}-nex_{NL})	If $mcp_{NL} - mcp_{DE} < \varepsilon$
LB:= max(LB,nex_{FR})	If $mcp_{DE} - mcp_{FR} < \varepsilon$
UB:= min(UB, 0)	If $mcp_{FR} - mcp_{BE} > \varepsilon$
UB:= min(UB,-nex_{BE})	If $mcp_{BE} - mcp_{NL} > \varepsilon$
UB:= min(UB,-nex_{BE}-nex_{NL})	If $mcp_{NL} - mcp_{DE} > \varepsilon$
UB:= min(UB,nex_{FR})	If $mcp_{DE} - mcp_{FR} > \varepsilon$

We start this process using ε=0.005€. If LB > UB the process is rerun, now using ε=0.025€.

We now set the optimal x* to:

```
if LB < UB {
    if x* < LB {x*←LB};
    if x* > UB {x*←UB};
} else {
    //stick with the initial x*, since it is not going to intuitive
    //anyway
}
```

The results bilateral exchanges become:

FR→BE:= max(x*, 0);

BE→FR:= max(-x*, 0);

BE→NL:= max(nex_{BE}+x*, 0);

NL→BE:= max(-nex_{BE}–x*, 0);

NL→DE:= max(nex_{BE}+nex_{NL}+x*, 0);

DE→NL:= max(-nex_{BE}-nex_{NL}-x*, 0);

DE→FR:= max(-nex_{FR}+x*, 0);

FR→DE:= max(nex_{FR}-x*, 0);

The justification for the perhaps rather abstract text in this section is provided in section 4.

3 Annex – restoring balance constraint

Due to rounding it is possible that the BEC calculation is provided with a series of net positions that do not sum to zero. Since deriving net positions from bilateral exchanges by definition result in a fully balanced system, it is important to restore the balance condition prior to deriving the corresponding exchanges.

The imbalance can be written as: $\sum_{z \in Z} nex_z = \Delta$

To restore balance, we adjust net positions in the direction of zero.

We define ordered sets:

$$Z^+ = \{z \in Z \mid nex_z > 0\}$$
$$Z^- = \{z \in Z \mid nex_z < 0\}$$

where elements are ordered according to descending $|nex_z|$.

We define Q_{tick} as the smallest nomination tick (e.g. 0.1MWh). Since Δ resulted from rounding, it by definition should correspond to a multiple of this nomination tick. We adjust net positions as follows (in pseudo code):

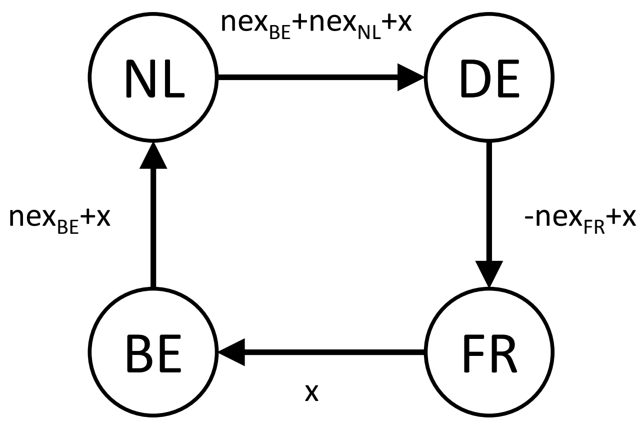
```
if Δ>0:
while (Δ>0) {
    for (z∈Z⁺){
        if (nex_z>0) {
            nex_z←nex_z-Q_tick
            Δ←Δ-Q_tick
        }
    }
}
if Δ<0:
while (Δ<0) {
    for (z∈Z⁺){
        if (nex_z<0) {
            nex_z←nex_z+Q_tick
            Δ←Δ+Q_tick
        }
    }
}
```

Note that the iterative fashion of the pseudo code is unnecessarily laborious, but this highlights that a net position should not be reduced past zero. More efficient implementations exist.

4 Annex – deriving bilateral exchanges

If we consider the CWE topology, it is clear that once the welfare maximizing net positions are known, the corresponding bilateral exchanges are not uniquely determined: for any set of bilateral exchanges, we can add an amount x to all exchanges, which is send in a loop, and consequently does not alter the net positions. I.e. any value of x gives another valid set of bilateral exchanges; hence the bilateral exchanges are not uniquely determined.

In the figure below we describe the bilateral exchanges by arbitrarily assign a value x to BE→FR. With BE→FR fixed, the other bilateral exchanges follow from the net positions:



Proposal unique solution I

If we fix x to a certain value, we have a unique set of bilateral exchanges. We propose to fix x, such that the sum of the squared bilateral exchanges is minimized. We can write the sum of the squared exchanges = f(x) as:

$$f(x) = x^2 + (nex_{BE} + x)^2 + (nex_{BE} + nex_{NL} + x)^2 + (-nex_{FR} + x)^2$$
$$= x^2 + \left[x^2 + 2nex_{BE}x + (nex_{BE})^2\right] + \left[x^2 + 2nex_{BE}x + 2nex_{NL}x + 2nex_{BE}nex_{NL} + (nex_{BE})^2 + (nex_{NL})^2\right] +$$
$$\left[x^2 - 2nex_{FR}x + (nex_{FR})^2\right]$$
$$= 4x^2 + (4nex_{BE} + 2nex_{NL} - 2nex_{FR})x + 2(nex_{BE})^2 + 2nex_{BE}nex_{NL} + (nex_{NL})^2 + (nex_{FR})^2$$

For this sum to be minimized, we need:

$$\frac{\partial f}{\partial x} = 0 = 8x + 4 \cdot nex_{BE} + 2 \cdot nex_{NL} - 2 \cdot nex_{FR}$$
$$\Rightarrow x^* = \frac{2nex_{BE} + nex_{NL} - nex_{FR}}{4}$$

Proposal unique solution II

The first solution does not consider any intuitive relation, i.e. the desire to find bilateral exchanges that go from a low priced area to a high priced area. Under FB "intuitive" we know such exchanges should exist. We derive the conditions that should be respected for a solution to be intuitive:

Imagine $mcp_{FR} < mcp_{BE}$. For a solution to be intuitive we need FR→BE ≥ 0, or x≥0. Conversely if $mcp_{FR} > mcp_{BE}$ we need x≤0. If we apply this logic on all 4 CWE exchanges we get:

$$FR \rightarrow BE : \begin{cases} x \geq 0 & \text{if } mcp_{FR} - mcp_{BE} < \varepsilon \\ x \leq 0 & \text{if } mcp_{FR} - mcp_{BE} > \varepsilon \end{cases}$$

$$BE \rightarrow NL : \begin{cases} x \geq -nex_{BE} & \text{if } mcp_{BE} - mcp_{NL} < \varepsilon \\ x \leq -nex_{BE} & \text{if } mcp_{BE} - mcp_{NL} > \varepsilon \end{cases}$$

$$NL \rightarrow DE : \begin{cases} x \geq -nex_{BE} - nex_{NL} & \text{if } mcp_{NL} - mcp_{DE} < \varepsilon \\ x \leq -nex_{BE} - nex_{NL} & \text{if } mcp_{NL} - mcp_{DE} > \varepsilon \end{cases}$$

$$DE \rightarrow FR : \begin{cases} x \geq nex_{FR} & \text{if } mcp_{DE} - mcp_{FR} < \varepsilon \\ x \leq nex_{FR} & \text{if } mcp_{DE} - mcp_{FR} > \varepsilon \end{cases}$$

Note we used a tolerance ε to account for the direction of the prices. Depending on the direction of the prices we establish different lower and upper bounds for x: LB ≤ x ≤ UB. If LB > UB no such x exists (or the tolerance was too tight), and the solution cannot be intuitive. If LB < UB, we select x according to:

If LB < x* < UB OR LB > UB[1] ⇒ x=x*

If x* < LB ⇒ x= LB;

If x* > UB ⇒ x=UB;

Where x* is the solution from I.

[1] Note that since we allow LB > UB, i.e. non-intuitive solution, this approach of computing BECs should also work when switching back to FB "plain" allocation.