

JAO eCAT

Report

Benders Decomposition Design

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1 Introduction

1.1 Context

The JAO capacity auctions allocate cross-border transmission capacity to market participants based on their bids. Historically, the auction clearing algorithm has operated **per hour** (Market Time Unit, MTU) and only with **single-MTU bids**. Each hour is treated independently, and the algorithm selects the set of bids that maximizes **social welfare** under capacity constraints.

Market development and participant needs have led to the introduction of **block bids**, which allow participants to express “all-or-nothing” demand for capacity across multiple hours. At the same time, the time resolution of the auction has evolved: from a 60-minute resolution to a finer **15-minute MTU** resolution at corridor level. To accommodate both legacy 60-minute products and new 15-minute products **on the same border**, the following design choice is adopted:

- The **underlying MTU resolution** for the optimization is **15 minutes**.
- Every “60-minute bid” is modeled as a **block bid spanning the four 15-minute intervals** within the same clock hour.
- Block bids are indivisible and cannot be partially accepted, so it must be accepted in full or rejected in full (“all-or-nothing”).
- As a result, what we will call **single-MTU bids** become, in implementation terms, **15-minute bids**, and **block bids** naturally represent:
 - 60-minute products (4 consecutive 15-minute MTUs within an hour), and
 - Longer multi-hour products (blocks spanning several 15-minute intervals across hours).

The inclusion of block bids significantly increases the complexity of the auction. This report explains how we integrate these block bids into the existing framework using **Benders decomposition**, in a way that:

- Maximizes overall welfare.
- Preserves consistency with existing hourly clearing.
- Respects business rules and fairness considerations.
- Remains computationally tractable for operational use.

This document explains the logic and implications of the chosen approach.

2 Current Auction Design: Single-MTU bids

2.1 Objective: Maximizing Social Welfare

In the current setup (without block bids), the auction is cleared **separately for each MTU**. For a given MTU, the algorithm:

- Receives a set of **single-MTU bids**.
- Each bid has:
 - A **source** and **sink** control area.
 - A **quantity** (MW) requested.
 - A **bid price** (€/MWh).

The algorithm's goal is to:

maximizes social welfare through a basic optimization subject to available transmission capacity and bid quantity limits, followed by a post-processing logic for “pro-rata”s.

The objective function consists of the following:

$$obj = \max \left(\sum_{x,y,b} [p_{b(x,y,b)} \cdot d_{a(x,y,b)}] \right)$$

2.2 Constraints

For each MTU, the main constraints are:

1. **Bid quantity constraints**
 - The accepted quantity for a bid cannot exceed the offered quantity.
 - Accepted quantity must be non-negative.
2. **Available Transfer Capacity (ATC) constraints**
 - The total net flow over each border cannot exceed the offered ATC for that MTU.
 - The formulation may vary depending on the technical profile (TP), but conceptually:
 - “Total flow using that border \leq ATC for that border and MTU.”

2.3 Auction price and Shadow prices

From this optimization, we derive **shadow prices** (dual variables) associated with ATC constraints. These shadow prices:

- Represent the **implicit marginal value of capacity** at a border and MTU.
- When aggregated for all relevant constraints on a given source–sink path, they determine the **auction price (AP)** for that path.

Two simple rules govern single-MTU bids:

- Bids with price **above the AP**: fully accepted.
- Bids with price **below the AP**: fully rejected.

This gives a transparent, rule-based outcome in each MTU and ensures that:

- Capacity is allocated to the highest-valued uses.
- The calculation of AP is consistent with the constraints and clearing outcome.

3 Introducing block bids

3.1 Business rules

To align block bids with the existing pricing framework on each time-interval, the TSOs defined the following rules:

1. All **single-MTU bids** with price **strictly higher than the auction price (AP)** are fully accepted.
2. All **single-MTU bids** with price **strictly lower than the auction price (AP)** are fully rejected.
3. All **block bids** with block price **strictly lower than the auction price (AP)** for at least one time-interval must be rejected.
4. All **block bids** with block price **strictly higher than the auction price (AP)** for all time-intervals can be rejected.

An improved pro-rata post processing algorithm will be applied per time-interval for all marginal single-MTU bids to treat the bids of same price fairly.

Marginal block bids will be subject to welfare maximization by being treated as standard candidates during the optimization.

3.2 Complexity induced by block bids

Block bids create two fundamental challenges:

1. **Inter-temporal linkage**
 - Decisions are no longer MTU-by-MTU: accepting a block ties together all MTUs in its block.

- A block might be profitable in some MTUs and unprofitable in others, but still beneficial overall.

2. All-or-nothing decisions

- Each block is represented by a binary decision: accepted (1) or rejected (0).
- The problem becomes a **Mixed-Integer Linear Program (MILP)**, which is significantly harder to solve than a single LP per MTU and cannot include the business rules logic in it.

Therefore, the **Benders decomposition** is used to retain the modular structure and leverage the existing clearing logic on each time-interval.

4 Conceptual Framework: bilevel perspective

Before describing Benders, it is useful to think of the problem as a **bilevel model**:

- **Upper level (leader)**: decides which block bids to accept (binary variables).
- **Lower level (follower)**: performs the clearing of single-MTU bids, for each MTU separately and simultaneously, given the remaining capacity after blocks are accepted.

The lower-level problem is the familiar:

- Per-MTU welfare maximization for single-MTU bids.
- Subject to ATC constraints and bid quantity constraints.

This lower-level problem is **convex (linear)** and satisfies **strong duality**, which means:

- The follower's optimal objective equals the optimal value of its dual problem.
- This property is a key enabler for Benders decomposition: it allows the use of dual (shadow price) information to guide the upper-level decisions.

5 Benders Decomposition

5.1.1 Basic principle

Benders decomposition splits the original complex problem into two linked pieces:

1. A master problem:

- Handles the **complicating variables**: here, the binary block acceptance decisions.
- Operates at a more aggregated level.

2. A **subproblem**:

- Handles the **continuous variables**: here, the single-MTU bids and flows.
- Retains the existing MTU-based resolution of the auction.

The method proceeds iteratively:

1. The master proposes a candidate set of block acceptances.
2. The subproblem clears the market for single-MTU bids under those choices.
3. Dual information from the subproblem is used to add a **Benders cut** to the master problem, refining its understanding of the welfare impact.
4. The master is re-solved with the new cut, proposing a new combination of blocks.
5. This repeats until convergence.

At convergence, the combination of block acceptances and clearing of single-MTU Bids on each MTU is **globally welfare-optimal** under the model and constraints.

5.1.2 Master problem (MILP, small)

The master problem is the “decision-maker” for block bids. It:

- Decides, for each block bid, whether it is **accepted (1)** or **rejected (0)**.
- Ensures that the sum of accepted block quantities on each border and MTU does not exceed the available ATC, so that the subproblem remains feasible.
- Estimates the resulting welfare from the **single-MTU bids**, without explicitly solving the full single-MTU bid problem inside the master.

To achieve this, the master problem includes:

- Binary variables indicating **block acceptance**.
- A *proxy variable*, called **θ (theta)**, representing:
 - “The best total welfare contribution from **all** single-MTU bids, given the capacity remaining after accepting these blocks.”
- Feasibility cuts that enforce basic capacity consistency:
 - For each border and MTU, the sum over all accepted blocks of (block quantity × acceptance binary) must be less than or equal to the ATC for that border and MTU.

The **master objective** is:

$$F_{master} = \max \left(\sum_{bb} \sum_{h \in H_b} p_{bb} \cdot d_{bb} \cdot y_{bb} + \theta \right)$$

5.1.2.1 Why use θ ?

θ is a **placeholder** for the subproblem's optimal welfare.

Initially, θ is unconstrained above (it could take any large value).

As we solve subproblems and collect information, we **add constraints (Benders cuts)** that lower the upper bound on θ , making it more accurate.

At the same time, the **feasibility cuts on block quantities vs. ATC** ensure that the master never proposes a block-acceptance pattern that would make the subproblem infeasible.

This feasibility cut in the master problem ensures that the sum of accepted block quantities never exceeds ATC on any border and MTU. Because the subproblem allows zero accepted quantity of single-MTU bids and does not include additional minimum-acceptance or intertemporal constraints, this is sufficient to guarantee that the subproblem is always feasible.

This keeps the master problem:

- **Relatively small:** mostly binary block variables plus θ .
- **Efficient and robust:** we avoid wasted iterations on subproblems that would fail due to impossible capacity over-commitment by blocks.

The master problem is structured to answer: “Which blocks should be accepted?” in a compact and feasible way, while treating the detailed clearing of single-MTU bids on each MTU through θ and Benders cuts. This separation makes the problem solvable at scale and allows adding business-specific “cuts” to deal with paradoxical block bids. Feasibility cuts ensure that the sum of accepted block quantities never exceeds ATC on any border and MTU, so the combined master + subproblem formulation remains feasible by construction.

5.1.3 Subproblem (LP, fast)

For a fixed block acceptance pattern (a fixed $y = \bar{y}$) and for each MTU:

- The subproblem receives the **remaining ATC per border and MTU**, after accounting for capacity used by accepted blocks.
- It then runs the **standard single-MTU bid auction LP**, for all MTUs separately:

- Maximizes welfare of single-MTU bids.
- Respects ATC and per-bid quantity constraints.

Its outputs include:

- The **optimal welfare contribution** from all single-MTU bids under the given block pattern.
- The **dual variables (shadow prices)** associated with the ATC and bid constraints.

The subproblem objective is the following:

$$F_{sub} = V(\bar{y}) = \max \left(\sum_b p_{b,h} \cdot d_{b,h} \right)$$

The problem is a standard single-MTU bid auction clearing LP. It is solved for each MTU separately and for all MTUs simultaneously.

5.1.3.1 Strong duality and shadow prices

The subproblem is a **linear program** with convex structure, so:

- It satisfies **strong duality**:
 - The primal objective (max welfare) equals the dual objective at optimum.
- The dual variables (shadow prices):
 - Reflect the marginal value of capacity on each border and MTU.
 - Are used to build **Benders cuts** for the master problem.

The subproblem precisely answers: “Given this set of accepted blocks, how can we best clear single-MTU bids and what is the value of remaining capacity?”

The associated shadow prices then tell the master how “expensive” block capacity usage is, so it can adjust its block decisions intelligently.

5.1.4 Benders Optimality Cut

After solving the subproblem for a given pattern of block acceptances:

- We know:
 - The actual welfare from single-MTU bids, given these blocks.
 - The shadow prices for ATC constraints.

From this, we construct a **Benders cut**: a linear inequality (Appendix A) that relates:

- The **block decisions** (which blocks are accepted).
- The **maximum welfare** that the subproblem can deliver (represented by θ).

This cut says:

“If you accept these blocks (and in general, blocks that use capacity in certain expensive MTUs), the best welfare you can get from single-MTU bids is no more than a certain value.”

Each cut provides a tighter upper bound on θ , helping the master problem approximate the true welfare impact of block decisions.

The key economic signals come from **shadow prices**:

- When capacity for a given MTU and border is very valuable (high shadow price), any block that uses this capacity is “expensive” to the system.
- The Benders cut reflects this by giving such blocks a more negative impact on θ whenever they consume capacity in those MTUs.

Thus:

- Blocks that consume capacity in **peak-value MTUs** get penalized more.
- Blocks that mainly use **low-value or uncongested MTUs** are less penalized.

Over successive iterations:

- The master learns that some blocks are systematically detrimental to welfare (due to high shadow prices at the times they use capacity) and tends to reject them.
- Other blocks that harmonize well with high-value single-MTU bids, or that fill low-value capacity, are more likely to be accepted.

θ shall be lower bounded either by 0 or by a certain value.

5.1.5 Iterative Process and Convergence

The Benders decomposition follows this loop:

1. Solve initial master

- Start with no Benders cuts.
- Include feasibility cuts that enforce:
 - For each border and MTU, the sum of (block quantity × acceptance binary) \leq ATC.

- θ is unconstrained above, so the master will initially tend to accept blocks that look valuable on their own (based on block price and quantity), since it “assumes” the single-MTU bid welfare can still be high.
- However, one can start from a neutral block acceptance which will provide the baseline welfare which is the maximum welfare one can get from the single-MTU bids without any block bids. Moreover, its duals (shadow prices) provide meaningful sensitivities. When no blocks are accepted, the shadow prices reflect the “pure” congestion value per MTU. So, if a MTU is tight (high shadow price), then the next iteration’s cut will penalize accepting blocks that consume capacity there. Also, it’s a conservative initialization that will measure how much value single-MTU bids produce, then progressively test block inclusion through cuts.
- Obtain:
 - Candidate block acceptance pattern y (which blocks are tentatively accepted).
 - Current θ value (estimated welfare from single-MTU bids).

Because of the feasibility cuts, any “ y ” chosen by the master already respects basic ATC limits across all block MTUs. This guarantees that the subproblem can find a feasible dispatch for single-MTU bids (possibly with zero accepted quantity if capacity is fully used by blocks).

2. Solve subproblem

- Fix y from the master.
- Adjust ATC per MTU and border based on accepted blocks (remaining capacity = ATC – block usage).
- Solve the single-MTU bid LP problem to optimality across all MTUs.
- Obtain:
 - Actual welfare from single-MTU bids.
 - Dual variables (shadow prices) for ATC constraints.

3. Compute Auction Price

- The Auction price is computed by choosing the cheapest accepted bid (single-MTU bid or block bid).
- This unifies the definition of the auction price across the master problem and the subproblem, since we cannot rely on the duals of the subproblem anymore.

4. Generate Cut

- Use strong duality and shadow prices to derive a Benders optimality cut.
- The cut limits θ as a function of block decisions and capacity usage.

5. **Apply the business rules**

- If, one of the business rules is violated:
 - Add a no-good cut forcing that block sequence to be rejected in subsequent iterations (e.g., $y_{\text{block}} = 0$).

6. **Update Master and re-solve**

- Add the new cut to the master problem.
- Any additional cuts from paradoxical block handling or other business rules.
- Feasibility cuts on block quantities vs. ATC remain in place throughout the process.
- Return to Step 1.
- Re-solve the master to obtain a new set of block decisions and a refined θ .

7. **Repeat until:**

- For the chosen block pattern, the θ in the master equals the true welfare from the subproblem.
- No new cuts further improve the solution.
- At this point, we have convergence.

8. **Apply tie-breaking algorithm:**

- If, more than one optimal solution is found, the tie-breaking algorithm is triggered to ensure that the solution with the least amount of block bids is chosen.

9. **Apply improved pro-rata algorithm:**

- If more than one single-MTU bid has equal bid prices, the improved pro-rata algorithm is triggered to equally allocate capacity.

The flowchart below visualizes the iterative process and the Benders Loop:

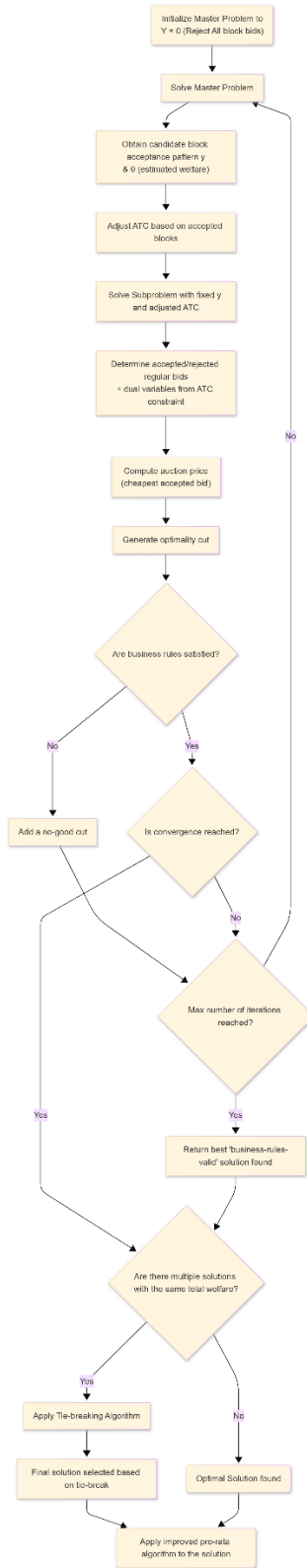


Figure 1. Algorithm Workflow.

5.1.6 Guaranteed maximum welfare

Because each Benders cut is derived from exact dual information of a convex LP:

- The master's upper bound on welfare becomes progressively tighter.
- The final solution is **globally optimal**:
 - There is no other combination of block acceptances and single-MTU bid dispatch that could yield higher total welfare under the modeled constraints.

In other words:

The iterative process systematically eliminates overly optimistic views of single-MTU welfare for each block combination, until it finds the combination for which expectations and reality match. That combination is the welfare-maximizing solution.

5.1.7 Fairness and non-Discrimination

5.1.7.1 Equal treatment in Welfare Objective

Both single-MTU bid and block bids are evaluated in the same welfare framework:

- Each bid's contribution is:
 - Quantity × price.
- The optimization allocates capacity to the combination of bids that gives the highest total welfare.

This ensures that:

- There is no explicit preference for one bid type over the other in the objective.
- Every euro of surplus is treated equally, regardless of its source.

5.1.7.2 Shadow Prices and Non-Discriminatory Capacity Valuation

Shadow prices (dual variables) provide a **uniform valuation of capacity**:

- All bids (block or single-MTU bid) face the **same shadow price** for using capacity on a given border and MTU.
- If capacity is scarce and valuable for a given period, both block and single-MTU bids are "charged" the same opportunity cost.

Thus:

- The optimization model does not discriminate between bid types in valuing capacity.
- Differences in outcomes stem from the structural differences:

- Single-MTU bids are flexible and can be accepted or rejected MTU by MTU.
- Block bids must be accepted or rejected in full, which can create paradoxical situations.

5.1.8 Block vs Single-MTU Bids: Are We Favouring One?

5.1.8.1 Methodological neutrality

From a model design perspective:

- The welfare objective treats all bids symmetrically.
- Capacity constraints, shadow prices, and the rules around AP apply consistently.

However, the **structural properties** of the bids imply differences:

- **Single-MTU bids:**
 - Have a simple and local rule relative to AP.
 - Each MTU is independent; a bid can be accepted in one MTU and rejected in another, even if from the same participant.
- **Block bids:**
 - Must be “all-or-nothing” across multiple MTUs.
 - May be rejected even if their price looks attractive in most individual MTUs (paradoxically rejected).
 - May be accepted even if some MTUs in the block are out of the money, as long as the **overall** impact is positive.

5.1.8.2 Perception of favouritism

In practice, this can lead to perceptions such as:

- “Block bids are harder to get accepted, even with a competitive price.”
- “Single-MTU bids always follow the simple AP rule, but blocks don’t.”

These differences are **not caused by Benders decomposition itself**, but by:

- The **all-or-nothing nature** of blocks.
- The requirement to maximize total welfare over multiple MTUs.

Therefore:

The algorithm does not intentionally favour one type of bid over another. It systematically chooses the combination of single-MTU and block bids that maximizes total welfare, but block bids are more constrained by design and hence more prone to paradoxical acceptance/rejection.

5.1.9 Handling the paradoxically accepted/rejected block bids

5.1.9.1 Paradoxically rejected blocks

A **paradoxically rejected** block is:

- A block that appears to be “in the money” based on observed auction prices but is rejected.
- This can occur because:
 - Accepting it would crowd out higher-value single-MTU bids or other blocks in one or more MTUs.
 - The welfare gain in some MTUs is outweighed by welfare losses in others.

Paradoxical rejections are an inherent feature of block products under welfare maximization with capacity constraints.

5.1.9.2 Paradoxically accepted blocks

A **paradoxically accepted** block would be:

- A block that, under the applied market prices, would not be profitable for the participant, yet is accepted.
- To avoid forcing participants into loss-making positions, business rule 3 (in Chapter 3.2) is enforced.

5.1.9.3 How does Benders Decomposition handle the paradoxical blocks

Benders decomposition alone does not resolve issues of paradoxical acceptance or rejection. It:

- Efficiently finds the welfare-maximizing solution under the given constraints.
- But does not, by itself, enforce revenue adequacy or “no loss” conditions for individual blocks.

Only paradoxically accepted blocks are going to be ruled out in the Benders Decomposition approach through business rule 3. To address these blocks **an integrated constraints approach** is followed. It consists of adding explicit constraints to the optimization that:

- Prevent paradoxical acceptances by bounding block revenue relative to prices.

- If a block is found paradoxical in an iteration, add a cut stating that this block must not be accepted.

5.1.10 Role of the business rules in Benders Decomposition

The business rules described in this report are not a technical necessity of Benders decomposition; they are a **deliberate market design choice**. Their purpose is to shape the welfare-maximizing outcome so that it aligns with JAO’s market principles and participants’ expectations.

In a purely mathematical sense, Benders decomposition would simply:

- Decide block acceptances in the master problem,
- Maximize welfare from single-MTU bids in the subproblem, and
- Iterate until overall welfare is maximized, without regard to how individual bids fare under the resulting prices.

What distinguishes our implementation is that we explicitly **embed business rules into the Benders loop**:

- Single-MTU bid rules (“bids strictly above AP fully accepted, bids strictly below AP fully rejected”) are enforced by rejecting the block-bid pattern that led to the violation of said rules. This is done by implementing a no-good cut such that:

$$\sum_{b \text{ in } B^*} (1 - y_b) \geq 1$$

Where B^* is the pattern of block bids that is examined at the iteration where either of the Single-MTU bid rules is being violated.

This no-good cut implies that: *“At least one block in the violating pattern must change state”*.

- Block bid rule 3 (“block price must not be strictly lower than the AP over its MTUs”) is checked at each iteration; any pattern that leads to a paradoxically accepted block is ruled out by adding a cut (e.g. forcing $y_b = 0$ for that block).
- Additional business-oriented criteria, such as tie-breaking rules between welfare-equivalent block patterns, are implemented at the master level (e.g. preferring solutions with more single-MTU bid welfare or fewer accepted blocks).

Benders decomposition is the enabling technical framework: it keeps the problem solvable at scale and exposes the right dual information (shadow prices) to build welfare-based cuts. The

business rules are what give this framework its market shape. They allow to “manipulate” the optimization in a controlled and transparent way:

- **Maximize total welfare** under physical and economic constraints, and at the same time
- **Enforce market rules** such as no-loss conditions for blocks, priority patterns for single-MTU bids, and consistent pricing logic.

5.1.11 Tie-breaking rules

In some configurations, the welfare-maximizing problem admits **multiple block-acceptance patterns with the same total welfare**. This is already visible in simple one-block examples:

- Example 1:
 - Rejecting the block ($y = 0$) and fully clearing the single-MTU bids yields total welfare = 400.
 - Accepting the block ($y = 1$) and crowding out the single-MTU bids also yields total welfare = 400.
- Example 4:
 - Rejecting the block ($y = 0$) and accepting the single-MTU bids in the first two MTUs yields total welfare = 400.
 - Accepting the block ($y = 1$) and crowding out the single-MTU bids yields the same 400.

From a pure welfare perspective, both patterns are equally optimal. However, in an operational implementation this ambiguity must be resolved in a **systematic and transparent way**, so that the algorithm always produces a unique outcome.

We therefore introduce **explicit tie-breaking rules** at the master-problem level, to select one block-acceptance pattern among welfare-equivalent alternatives.

5.1.11.1 Design choice

Different tie-breaking philosophies are possible. Two examples can be found below:

1. **Preference for single-MTU bids:**
Among all welfare-maximizing solutions, prefer those that keep as much welfare as possible in single-MTU bids. Intuitively, blocks are used only when they clearly bring additional welfare beyond what the single-MTU bids can provide.
2. **Preference for fewer accepted blocks (minimal block usage):**
Among all welfare-maximizing solutions, prefer solutions with the smallest number of

accepted blocks. This keeps the outcome closer to the legacy single-MTU bid world and reduces the number of all-or-nothing commitments.

Both philosophies lead to the same choice in Examples 1 and 4 (rejecting the block), but they are conceptually distinct and could differ in more complex situations.

The TSOs selected the second rule: Preference for fewer accepted blocks (minimal block usage)

5.1.11.2 Implementation

We implement tie-breaking as a lexicographic optimization on top of the Benders master problem. The procedure is as follows:

1. The standard Benders decomposition is executed until convergence, yielding the maximum achievable total welfare W^* .
2. All Benders cuts and feasibility constraints identified during the convergence process are retained.
3. The master problem is **re-solved** with:

- An additional constraint fixing total welfare at its optimum: W^*

$$\sum_{b \in B} W_b y_b + \theta \geq W^*$$

- A new objective function that **minimizes the number of accepted block bids**:

$$\min \left(\sum_b y_b \right)$$

Because this second step operates on the already converged master problem and does not alter the subproblem structure, **no additional subproblem solves are required**.

Among all welfare-maximizing block patterns, we pick the one with the smallest number of accepted blocks.

In case more than 1 solution has the same number of accepted block bids, the solution with the block bid that has the lowest ID number is chosen.

5.1.12 Improved pro rata algorithm

To allocate capacity fairly for all single-MTU bids having the same bid prices and the same Source-Sink Pair for one time-interval (and the single-MTU bids cannot be fully accepted), an improved pro rata algorithm is applied to the single-MTU bids for every time-interval separately.

The improved pro rata algorithm is run after the tie-breaking algorithm and its principle consists of an equal allocation of the remaining capacity to the respective market participants according to number of bids with the same bid price:

- If two (2) or more registered participants have submitted for one technical profile with valid bids and the same bid price, that cannot be fully accepted for a total requested quantity of rights, improved pro-rata approach is applied to determine winning bids and quantity of allocated rights with regard these bids and registered participants as follows:
 1. Remaining capacity which can be shared between the respective registered participants is divided by number of such registered participants and rounded down to nearest full MW.
 2. In case the requested quantity of rights by registered participant is lower or equal than the share calculated according to bullet 1), the request of registered participant is fully satisfied.
 3. In case the requested quantity of rights by registered participant exceeds the share calculated according to bullet 1) the request of registered participant is satisfied in amount calculated according to bullet 1) above.
 4. The remaining capacity, which can be shared between the respective registered participants, is recalculated, i.e. decreased by amounts allocated according to bullet 2) and 3).
 5. Procedure described in the bullet 1 to 5 is repeated with registered participants mentioned under bullet 3), i.e. whose request is only partially satisfied.

6 Examples

This section contains examples of corner cases to see how Benders Decomposition will tackle them:

6.1 Example 1

MTU	ATC	MP1 (single-MTU bids)	MP2 (block bid)
10:00–10:15	10	10 MW @ 15 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	10 MW @ 10 €/MWh	
10:30–10:45	10	10 MW @ 10 €/MWh	
10:45–11:00	10	10 MW @ 5 €/MWh	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

6.1.1 Iteration 1: baseline scenario

If we decide to initialize the Benders decomposition to $y=0$, the block bid is initially rejected. The ATC is fully available in each MTU for single-MTU bids.

Subproblem with $y=0$:

- $t=1$: accept 10 @15 → welfare 150, ATC used.
- $t=2$: accept 10 @10 → welfare 100, ATC used.
- $t=3$: accept 10 @10 → welfare 100.
- $t=4$: accept 10 @5 → welfare 50.

So:

- $V(0) = 150 + 100 + 100 + 50 = 400$.
- $\lambda = (0, 0, 0, 0)$.

The auction price is equal to the cheapest accepted bid (when there is no ATC slack), so $AP = (15, 10, 10, 5)$.

Business rules check:

All the accepted Single-MTU bids are higher than or equal to the auction price, and the price of the rejected block bid is strictly lower than the auction price on the first time-interval so the solution is valid.

Optimality cut from iteration 1:

$$\theta \leq 400$$

Master moves to iteration 2 and accepts the block bid.

6.1.2 Iteration 2: block bid accepted ($y = 1$)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $\theta \leq 400$
2. $10y \leq 10$
3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with $y = 1$:
 - The remaining capacity for single-MTU bids = 0 MW in all 4 MTUs
 - No capacity left, so no MP1 bids can be accepted.
 - Single-MTU bid welfare: $V(1) = 0$.
 - $\lambda = (15, 10, 10, 5)$.

The auction price is equal to the cheapest accepted bid so $AP = (10, 10, 10, 5)$.

Business rules check:

The price of the rejected Single-MTU bid is strictly higher than the auction price on the first time-interval, so Business Rule 1 is violated, which means that the solution is not valid.

Therefore:

We force-reject the block bid pattern by implementing a no-good cut:

$$\sum_{b \text{ in } B^*} (1 - y_b) \geq 1$$

Where B^* is the pattern of block bids that is examined at this iteration.

In the context of this example, only one block bid exists therefore the no-good cut implies that y_b should not be equal to 1 and the block bid should be rejected.

Optimality cut from iteration 2:

$$\theta \leq 400 - 400 y$$

We substitute the cut:

- For $y=0$: $\theta \leq 400 \rightarrow$ best θ is 400 \rightarrow objective = $0 \cdot 400 + 400 = 400$.

Converged solution: $y=0$, total welfare 400.

6.1.3 Auction price computation

MTU	Shadow price	AP (€/MWh)
10:00–10:15	0	15

MTU	Shadow price	AP (€/MWh)
10:15–10:30	0	10
10:30–10:45	0	10
10:45–11:00	0	5

6.1.4 Final auction results

MTU	ATC (MW)	MP1 (single-MTU bid) selected quantity (MW)	MP2 (block bid) selected quantity (MW)	Auction Price (€/MWh)
10:00–10:15	10	10	0	15
10:15–10:30	10	10	0	10
10:30–10:45	10	10	0	10
10:45–11:00	10	10	0	5

6.1.5 Results interpretation

Welfare is identical in both cases, so the Benders master problem has at least two optimal solutions ($y = 0$ or $y = 1$) from a pure welfare perspective.

Detailed Benders interpretation

- Subproblem with $y=1$ gives actual welfare from single-MTU bids = 0 and duals on ATC indicate that marginal value of capacity would be around 15, 10, 10, 5 €/MWh in the corresponding MTUs if capacity were available.
- Benders cut basically says:
“If you allocate all capacity to this block, the best welfare from single-MTU bids is 0 and

the opportunity cost is exactly equal to the value you would have obtained with single-MTU bids.”

- Therefore, total welfare is 400 either way.

Business Rule Validation

- For $y=0$ (block rejected):
 - The block passes Business Rule 3 but would be rejected based on tie-breaking preference
- For $y=1$ (block accepted):
 - **Business Rule 1 Violation:** The single-MTU bid in MTU 1 has price 15 €/MWh > AP (10 €/MWh)
 - According to Rule 1: "All single-MTU bids with price higher than AP are fully accepted"
 - This bid **must** be accepted, but there is no capacity available
 - **Therefore, solution $y=1$ is invalid.**

Final Decision

- Without business rules, the implementation would pick one solution depending on the tie-breaking algorithm put in place (likely $y=0$ based on preference for fewer accepted blocks).
- However, with business rule enforcement:
 - $y=1$ is excluded due to Business Rule 1 violation
 - A constraint forcing $y=0$ is added to the master problem

Key Takeaway

This example demonstrates that welfare maximization alone is insufficient. Even though both solutions yield welfare = 400, accepting the block would violate the fundamental market rule that bids priced strictly above the auction price must be accepted. The business rules transform what would be an arbitrary tie-breaking decision into a definitive outcome that preserves market consistency

6.2 Example 2

MTU	ATC	MP1	MP2 (block)
10:00–10:15	10	10 MW @ 14 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	10 MW @ 10 €/MWh	
10:30–10:45	10	10 MW @ 10 €/MWh	
10:45–11:00	10	10 MW @ 5 €/MWh	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

- Feasibility cut (block_quantity <= ATC): $10y \leq 10$
- $y \in \{0; 1\}$

6.2.1 Iteration 1: baseline scenario

The block bid is initially rejected. The ATC is fully available in each MTU for single-MTU bids.

Subproblem with $y=0$:

- $t=1$: accept 10 @15 → welfare 150, ATC used.
- $t=2$: accept 10 @10 → welfare 100, ATC used.
- $t=3$: accept 10 @10 → welfare 100.
- $t=4$: accept 10 @5 → welfare 50.

So:

- $V(0) = 140 + 100 + 100 + 50 = 390$.
- $\lambda = (0, 0, 0, 0)$.

The auction price is equal to the cheapest accepted bid so $AP = (15, 10, 10, 5)$.

Business rules check:

The solution is valid.

Optimality cut from iteration 1:

$$\theta \leq 390$$

Master moves to iteration 2 and accepts the block bid.

6.2.2 Iteration 2: block accepted ($y = 1$)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $\theta \leq 390$
2. $10y \leq 10$
3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with $y = 1$:
 - The remaining capacity for single-MTU bids = 0 MW in all 4 MTUs
 - No capacity left, so no MP1 bids can be accepted.
 - Single-MTU bid welfare: $V(1) = 0$.
 - $\lambda = (14, 10, 10, 5)$.

The auction price is equal to the cheapest accepted bid so $AP = (10, 10, 10, 10)$.

Business rules check:

The price of the rejected Single-MTU bid is strictly higher than the auction price on the first time-interval, so Business Rule 1 is violated, which means that the solution is not valid.

Therefore:

We force-reject the block bid pattern by implementing a no-good cut:

$$\sum_{b \in B^*} (1 - y_b) \geq 1$$

Where B^* is the pattern of block bids that is examined at this iteration.

In the context of this example, only one block bid exists therefore the no-good cut implies that y_b should not be equal to 1 and the block bid should be rejected.

Optimality cut from iteration 2:

$$\theta \leq 0 - 390 (y - 1)$$

$$\theta \leq 390 - 390 y$$

Objective:

- $y=0 \rightarrow 400 \cdot 0 + \theta \leq 390 = 390$.
- $y=1 \rightarrow 400 \cdot 1 + \theta \leq 400 + 0 = 400$.

Therefore, due to the business rules being violated the master chooses to reject the block bid.

$y^* = 0$ (block rejected), total welfare = 390 (even if higher welfare is reached when the block bid is accepted).

6.2.3 Auction price computation

The auction price is equal to the cheapest accepted bid which in this case is the Single-MTU bid for each MTU.

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	0	14
10:15–10:30	0	10
10:30–10:45	0	10
10:45–11:00	0	5

6.2.4 Final auction results

MTU	ATC (MW)	MP1 (single-MTU bid) selected quantity (MW)	MP2 (block bid) selected quantity (MW)	Auction Price (€/MWh)
10:00–10:15	10	10	0	14
10:15–10:30	10	10	0	10

MTU	ATC (MW)	MP1 (single-MTU bid) selected quantity (MW)	MP2 (block bid) selected quantity (MW)	Auction Price (€/MWh)
10:30–10:45	10	10	0	10
10:45–11:00	10	10	0	5

6.2.5 Results interpretation

Welfare is higher when the block is accepted:

- Welfare ($y=1$) = 400
- Welfare ($y=0$) = 390
→ block **increases welfare** by 10 compared to only single-MTU bids.

From the Benders decomposition angle:

- First iteration: Subproblem with $y=0$ yields welfare 390; dual prices show that capacity is especially valuable in the first interval (14 €/MWh).
- Evaluating $y=1$, subproblem gives 0 welfare from single-MTU bids: total = 400 via block.
- Master receives a cut indicating that using capacity for the block is beneficial — θ under “block accepted” can be higher in combination with the block’s own contribution.

Business Rule Validation

- For $y=0$ (block rejected):
 - All business rules are validated.
- For $y=1$ (block accepted):
 - **Business Rule 1 Violation:** The single-MTU bid in MTU 1 has price 14 €/MWh > AP (10 €/MWh)
 - According to Rule 1: "All single-MTU bids with price higher than AP are fully accepted"
 - This bid **must** be accepted, but there is no capacity available
 - **Therefore, solution $y=1$ is invalid.**

Final Decision

- Without business rules, the implementation would pick the solution that has the higher welfare ($y=1$).
- However, with business rule enforcement:
 - $y=1$ is excluded due to Business Rule 1 violation
 - A constraint forcing $y=0$ is added to the master problem

6.3 Example 3

MTU	ATC	MP1 (single-MTU bid)	MP2 (block)
10:00–10:15	10	10 MW @ 15 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	— (no bid)	
10:30–10:45	10	— (no bid)	
10:45–11:00	10	— (no bid)	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

- Feasibility cut (block_quantity \leq ATC): $10y \leq 10$
- $y \in \{0; 1\}$

6.3.1 Iteration 1: baseline scenario

The block bid is initially rejected. The ATC is fully available in each MTU for single-MTU bids.

Subproblem with $y=0$:

- t=1: accept 10 @15 \rightarrow welfare 150, ATC used.
- t=2: $\rightarrow \lambda_2 = 0$.
- t=3: $\rightarrow \lambda_3 = 0$.

- $t=4: \rightarrow \lambda_4 = 0$.

So:

- $V(0) = 150$.
- $\lambda = (0, 0, 0, 0)$.

The auction price is equal to the cheapest accepted bid so AP = 15 on the first time-interval.

Business rules check:

The solution is valid.

Optimality cut from iteration 1:

$$\theta \leq 150$$

Master moves to iteration 2 and accepts the block bid.

6.3.2 Iteration 2: block accepted ($y = 1$)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $\theta \leq 150$
2. $10y \leq 10$
3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with $y = 1$:
 - The remaining capacity for single-MTU bids = 0 MW in all 4 MTUs
 - No capacity left, so no MP1 bids can be accepted.
 - Single-MTU bid welfare: $V(1) = 0$.
 - $\lambda = (15, 0, 0, 0)$.

The auction price is equal to the cheapest accepted bid so AP = (10, 10, 10, 10).

Business rules check:

The price of the rejected Single-MTU bid is strictly higher than the auction price on the first time-interval, so Business Rule 1 is violated, which means that the solution is not valid.

Therefore:

We force-reject the block bid pattern by implementing a no-good cut:

$$\sum_{b \text{ in } B^*} (1 - y_b) \geq 1$$

Where B^* is the pattern of block bids that is examined at this iteration.

In the context of this example, only one block bid exists therefore the no-good cut implies that y_b should not be equal to 1 and the block bid should be rejected.

Optimality cut from iteration 2:

$$\theta \leq 0 - 150 (y - 1)$$

$$\theta \leq 150 - 150 y$$

We substitute the cut:

- For $y=0$: $\theta \leq 150 \rightarrow$ best θ is 150 \rightarrow objective = $0 \cdot 150 + 150 = 150$.
- For $y=1$: $\theta \leq 150(1-1) = 0 \rightarrow$ best θ is 0 \rightarrow objective = $400 \cdot 1 + 0 = 400$.

Therefore, due to the business rules being violated the master chooses to reject the block bid.

$y^* = 0$ (block rejected), total welfare = 150 (even if higher welfare is reached when the block bid is accepted).

6.3.3 Auction price computation

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	0	15
10:15–10:30	0	0
10:30–10:45	0	0
10:45–11:00	0	0

6.3.4 Final auction results

MTU	ATC (MW)	MP1 (single-MTU bid) selected quantity (MW)	MP2 (block bid) selected quantity (MW)	Auction Price (€/MWh)
10:00–10:15	10	10	0	15
10:15–10:30	10	-	0	0
10:30–10:45	10	-	0	0
10:45–11:00	10	-	0	0

6.3.5 Results interpretation

Welfare is higher when the block is accepted:

- Welfare ($y=1$) = 400
- Welfare ($y=0$) = 150
→ block **massively increases welfare** compared to only single-MTU bids.

From a Benders Decomposition perspective:

- Master tries $y=0$ first.
- Subproblem yields $\theta=150$, with strong dual in first interval and slack in others.
- Master then tries $y=1$:
 - Subproblem welfare from single-MTU bids = 0.
 - But adding block's own welfare (400) yields total of 400.
- The Benders cut makes it clear that capacity in the three intervals with no MP1 bids is essentially free capacity; using it for a block adds pure welfare with no opportunity cost.
- The single-MTU bid in the first MTU is crowded out by the block because the block's additional welfare in the other 3 MTUs compensates the 150.

Business Rule Validation

- For $y=0$ (block rejected):

- All business rules are validated.
- For $y=1$ (block accepted):
 - **Business Rule 1 Violation:** The single-MTU bid in MTU 1 has price 15 €/MWh > AP (10 €/MWh)
 - According to Rule 1: "All single-MTU bids with price higher than AP are fully accepted"
 - This bid **must** be accepted, but there is no capacity available
 - **Therefore, solution $y=1$ is invalid.**

Final Decision

- Without business rules, the implementation would pick the solution that has the higher welfare ($y=1$).
- However, with business rule enforcement:
 - $y=1$ is excluded due to Business Rule 1 violation
 - A (no-good cut) constraint forcing $y=0$ is added to the master problem.

6.4 Example 4

MTU	ATC	MP1 (single-MTU bid)	MP2 (block)
10:00–10:15	10	10 MW @ 20 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	10 MW @ 20 €/MWh	
10:30–10:45	10	—	
10:45–11:00	10	—	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

- Feasibility cut (block_quantity \leq ATC): $10y \leq 10$
- $y \in \{0; 1\}$

6.4.1 Iteration 1: baseline scenario

The block bid is initially rejected. The ATC is fully available in each MTU for single-MTU bids.

Subproblem with $y=0$:

- $t=1$: accept 10 @20 \rightarrow welfare 200, ATC used.
- $t=2$: accept 10 @20 \rightarrow welfare 200, ATC used.
- $t=3$: $\rightarrow \lambda_3 = 0$.
- $t=4$: $\rightarrow \lambda_4 = 0$.

So:

- $V(0) = 400$.
- $\lambda = (0, 0, 0, 0)$.

The auction price is equal to the cheapest accepted bid so $AP = 20$ on the two first time intervals.

Business rules check:

The solution is valid.

Optimality cut from iteration 1:

$$\theta \leq 400$$

Master moves to iteration 2 and accepts the block bid.

6.4.2 Iteration 2: block accepted ($y = 1$)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $\theta \leq 400$
2. $10y \leq 10$
3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with $y = 1$:
 - The remaining capacity for single-MTU bids = 0 MW in all 4 MTUs
 - No capacity left, so no MP1 bids can be accepted.

- Single-MTU bid welfare: $V(1) = 0$.
- $\lambda = (20, 20, 0, 0)$.

The auction price is equal to the cheapest accepted bid so AP =10 for all MTUs.

Business rules check:

The price of the rejected Single-MTU bid is strictly higher than the auction price on the first time-interval, so Business Rule 1 is violated, which means that the solution is not valid.

Therefore:

We force-reject the block bid pattern by implementing a no-good cut:

$$\sum_{b \in B^*} (1 - y_b) \geq 1$$

Where B^* is the pattern of block bids that is examined at this iteration.

In the context of this example, only one block bid exists therefore the no-good cut implies that y_b should not be equal to 1 and the block bid should be rejected.

Optimality cut from iteration 2:

$$\theta \leq 0 - 400 (y - 1)$$

$$\theta \leq 400 - 400 y$$

We substitute the cut:

- For $y=0$: $\theta \leq 400 \rightarrow$ best θ is 400 \rightarrow objective = $0 \cdot 400 + 400 = 400$.
- For $y=1$: $\theta \leq 400 (1-1) = 0 \rightarrow$ best θ is 0 \rightarrow objective = $400 \cdot 1 + 0 = 400$.

Therefore, due to the business rules being violated the master chooses to reject the block bid.

$y^* = 0$ (block rejected), total welfare = 400

6.4.3 Auction price computation

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	0	20
10:15–10:30	0	20

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:30–10:45	0	0
10:45–11:00	0	0

6.4.4 Final auction results

MTU	ATC (MW)	MP1 (single-MTU bid) selected quantity (MW)	MP2 (block bid) selected quantity (MW)	Auction Price (€/MWh)
10:00–10:15	10	10	0	20
10:15–10:30	10	10	0	20
10:30–10:45	10	-	0	0
10:45–11:00	10	-	0	0

6.4.5 Results interpretation

Welfare is identical in both cases, so the Benders master problem has at least two optimal solutions ($y = 0$ or $y = 1$) from a pure welfare perspective.

Detailed Benders interpretation

- Subproblem with $y=1$ gives actual welfare from single-MTU bids = 0 and duals on ATC indicate that marginal value of capacity would be around 15, 10, 10, 5 €/MWh in the corresponding MTUs if capacity were available.
- Benders cut basically says:
“If you allocate all capacity to this block, the best welfare from single-MTU bids is 0 and the opportunity cost is exactly equal to the value you would have obtained with single-MTU bids.”

Business Rule Validation

- For $y=0$ (block rejected):
 - All business rules are validated.
- For $y=1$ (block accepted):
 - **Business Rule 1 Violation:** The single-MTU bid in MTU 1 has price 20 €/MWh > AP (10 €/MWh)
 - According to Rule 1: "All single-MTU bids with price higher than AP are fully accepted"
 - This bid **must** be accepted, but there is no capacity available
 - **Therefore, solution $y=1$ is invalid.**

Final Decision

- Without business rules, the implementation would pick the solution that has the higher welfare ($y=1$).
- However, with business rule enforcement:
 - $y=1$ is excluded due to Business Rule 1 violation
 - A constraint forcing $y=0$ is added to the master problem

6.5 Example 5

MTU	ATC	MP1 (single-MTU bid)	MP2 (block)
10:00–10:15	10	2 MW @ 20 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	2 MW @ 20 €/MWh	
10:30–10:45	10	—	
10:45–11:00	10	—	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

- Feasibility cut (block_quantity <= ATC): $10y \leq 10$
- $y \in \{0; 1\}$

6.5.1 Iteration 1: baseline scenario

The block bid is initially rejected. The ATC is fully available in each MTU for single-MTU bids.

Subproblem with $y=0$:

- $t=1$: accept 2 @20 → welfare 40, ATC used.
- $t=2$: accept 2 @20 → welfare 40, ATC used.
- $t=3$: → $\lambda_3 = 0$.
- $t=4$: → $\lambda_4 = 0$.

So:

- $V(0) = 80$.
- $\lambda = (0, 0, 0, 0)$.

The auction price is equal to the cheapest accepted bid so AP = 20 on the two first time intervals.

Business rules check:

All the accepted Single-MTU bids are strictly higher than the auction price, so the solution is valid.

Optimality cut from iteration 1:

$$\theta \leq 80$$

Master moves to iteration 2 and accepts the block bid.

6.5.2 Iteration 2: block accepted ($y = 1$)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $0 \leq \theta \leq 80$
2. $10y \leq 10$
3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with $y = 1$:
 - The remaining capacity for single-MTU bids = 0 MW in all 4 MTUs
 - No capacity left, so no MP1 bids can be accepted.
 - Single-MTU bid welfare: $V(1) = 0$.
 - $\lambda = (20, 20, 0, 0)$.

The auction price is equal to the cheapest accepted bid so $AP = (10, 10, 0, 0)$.

Optimality cut from iteration 2:

$$\theta \leq 400 - 400 y$$

$\theta \leq 80$ (optimality cut generated in iteration 1)

We substitute the cut:

- For $y=0$: $\theta \leq \min(400; 80) \rightarrow$ best θ is 80 \rightarrow objective = $0 \cdot 400 + 80 = 80$.
- For $y=1$: $\theta \leq \min(0; 80) = 0 \rightarrow$ best θ is 0 \rightarrow objective = $1 \cdot 400 + 0 = 400$.

Business rules check:

The price of the rejected Single-MTU bid is strictly higher than the auction price on the first time-interval, so Business Rule 1 is violated, which means that the solution is not valid.

Therefore:

We force-reject the block bid pattern by implementing a no-good cut:

$$\sum_{b \text{ in } B^*} (1 - y_b) \geq 1$$

Where B^* is the pattern of block bids that is examined at this iteration.

In the context of this example, only one block bid exists therefore the no-good cut implies that y_b should not be equal to 1 and the block bid should be rejected.

Therefore, due to the business rules being violated the master chooses to reject the block bid.

$y^* = 0$ (block rejected), total welfare = 80 (even if higher welfare is reached when the block bid is accepted).

6.5.3 Auction price computation

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	0	20
10:15–10:30	0	20
10:30–10:45	0	0
10:45–11:00	0	0

6.5.4 Final auction results

MTU	ATC (MW)	MP1 (single-MTU bid) selected quantity (MW)	MP2 (block bid) selected quantity (MW)	Auction Price (€/MWh)
10:00–10:15	10	2	0	20
10:15–10:30	10	2	0	20
10:30–10:45	10	-	0	0
10:45–11:00	10	-	0	0

6.6 Example 6

MTU	ATC	MP1	MP2	MP3
10:00–10:15	25	10 MW @ 20 €/MWh	Block 10 MW @ 10 €/MWh	Block 10 MW @ 10 €/MWh

MTU	ATC	MP1	MP2	MP3
10:15– 10:30	25	10 MW @ 20 €/MWh		
10:30– 10:45	25	—		
10:45– 11:00	25	—		

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y_2 + 400 \cdot y_3 + \theta)$$

Constraints:

- Feasibility cut (block_quantity <= ATC): $10y_2 + 10y_3 \leq 25$
- $y_2, y_3 \in \{0; 1\}$

Feasibility (capacity):

$10y_2 + 10y_3 \leq 25$ in each MTU; so the constraint in master is $10y_2 + 10y_3 \leq 25$ (MTUs 3–4) and $10y_2 + 10y_3 + 10$ (MP1) ≤ 25 in MTUs 1–2, but **MP1 is not in the master**, so the master only imposes:

- $10y_2 + 10y_3 \leq 25$ (blocks alone not exceeding ATC).

Hence $(y_2, y_3) = (1, 1)$ is *feasible* in master; the capacity conflict with MP1 is discovered via duals in the subproblem.

6.6.1 Iteration 1: baseline scenario

The block bids are initially rejected. The ATC is fully available in each MTU for single-MTU bids.

Subproblem with $y_2 = y_3 = 0$:

- t=1: accept 10 @20 → welfare 200, ATC not fully used (capacity slack), $\lambda_1 = 0$.
- t=2: accept 10 @20 → welfare 200, ATC not fully used (capacity slack), $\lambda_2 = 0$.
- t=3: → $\lambda_3 = 0$.
- t=4: → $\lambda_4 = 0$.

So:

- $V(0) = 400$.
- $\lambda = (0, 0, 0, 0)$.

The auction price is equal to the cheapest accepted bid so AP = 20 on the two first time intervals.

Optimality cut from iteration 1:

$$\theta \leq 400$$

The first cut is simply: $\theta \leq 400$ (no dependence on y_2 and y_3).

Master moves to iteration 2.

6.6.2 Iteration 2: blocks accepted (1, 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y_2 + 400 \cdot y_3 + \theta)$$

Constraints:

1. $0 \leq \theta \leq 400$
2. $10y_2 + 10y_3 \leq 25$
3. $y \in \{0; 1\}$

To maximize, master sets $\theta=400$ and $y_2=y_3=1$, because there's no penalty yet. Objective:

$$400 \cdot 1 + 400 \cdot 1 + 400 = 1,200.$$

Therefore, the master proposes (1,1).

Subproblem with $y_2=y_3=1$ and rem. ATC=5:

- t=1: accept 5 @20 → welfare 100, ATC used, $\lambda_1 = 20$.
- t=2: accept 5 @20 → welfare 100, ATC used, $\lambda_2 = 20$.
- t=3: → $\lambda_3 = 0$.
- t=4: → $\lambda_4 = 0$.

So:

- $V(1,1) = 200$.
- $\lambda = (20, 20, 0, 0)$.

Optimality cut from iteration 2:

$$\theta \leq 200 - 400(y_2 - 1) - 400(y_3 - 1)$$

$$\theta \leq 1000 - 400(y_2 + y_3)$$

Now the master is constrained by:

- $\theta \leq 400$.
- $\theta \leq 1000 - 400(y_2 + y_3)$.

(0,0):

- $\theta \leq 400$ and $\theta \leq 1000 - 0 = 1000 \rightarrow \theta \leq 400$
- $\text{Obj} = 0 + \theta \leq 400 \Rightarrow \text{best} = 400$.

(1,0) or (0,1): say (1,0):

- $\theta \leq 400$ and $\theta \leq 600 \Rightarrow \theta \leq 400$
- $\text{Obj} = 400 \cdot 1 + 400 \cdot 0 + \theta \leq 400 + 400 = 800 \Rightarrow \text{best} = 800$.

(1,1):

- $\theta \leq 400$ and $\theta \leq 200 \Rightarrow \theta \leq 200$
- $\text{Obj} = 400 \cdot 1 + 400 \cdot 1 + \theta \leq 800 + 200 = 1000$.

Master still prefers (1,1) with $\theta = 200 \Rightarrow$ objective 1,000

Benders selects $y^* = (1,1)$ (blocks accepted), $\theta^* = 200$, total welfare = $400 + 400 + 200 = 1000$

6.6.3 Applying business rules

If business rules 1, 2, 3 and 4 were to be taken into consideration by the Benders Decomposition:

6.6.3.1 Pattern A: $(y_2, y_3) = (0,0)$

No blocks accepted.

- Same as iteration 1:
 - t=1: MP1 10 MW @20 \rightarrow 200.
 - t=2: MP1 10 MW @20 \rightarrow 200.
 - t=3,4: 0.

- Total welfare: $W_{\text{blocks}}(0,0) + V(0,0) = 0 + 400 = 400$

6.6.3.2 Pattern B: $(y_2, y_3) = (1,0)$ or $(0,1)$ — exactly one block accepted

Remaining ATC per MTU = 15 MW

Subproblem, enforcing business rules:

- $t=1,2$:
 - Remaining ATC = 15.
 - MP1 wants 10 MW @20.
 - Capacity ≥ 10 , so MP1 must be fully accepted at 10 MW.
 - No other single-MTU bids, so:
 - MP1 = 10 MW, block MP2 = 10 MW (fixed), total $20 \leq 25$.
 - Welfare $t=1$ and $t=2$:
 - MP1: $10 \times 20 = 200$ each.
 - Block: $10 \times 10 = 100$ each.
 - Total per MTU = 300.
- $t=3,4$:
 - No single-MTU bids, but block uses 10 MW.
 - Welfare per MTU = block $10 \times 10 = 100$.

Total welfare = **800**.

Pattern $(0,1)$ gives the same total welfare by symmetry.

6.6.3.3 Pattern C: $(y_2, y_3) = (1,1)$ — both blocks accepted

Now each block uses 10 MW per MTU, so total block usage per MTU = 20 MW.

Remaining ATC per MTU = 5 MW

Subproblem:

- $t=1,2$:
 - Remaining ATC = 5.
 - MP1 bid is 10 MW @20.

- Business rule: **MP1 must be fully accepted** because its bid price is strictly higher than the auction Price (AP = 10).
- But here capacity for the single-MTU bid is only 5 MW, **not enough** to accept 10 MW.
- That means we **must** accept MP1 and reject one of the block bids.
- t=3,4:
 - Still 5 MW residual ATC, but no single-MTU bids.

This block-acceptance pattern violates **Business Rule 1**, which requires that all single-MTU bids priced above AP be fully accepted. Therefore, the configuration **(1,1) is infeasible** and must be excluded from the solution space.

Total welfare:

- MP2: $10 \times 10 \times 4 = 400$.
- MP3: $10 \times 10 \times 4 = 400$.
- MP1: 0.

Total = **800**.

So, both patterns (1,0); (0,1) and (1,1) produce **800** total welfare. But:

- The pure welfare-maximization problem admits multiple solutions with welfare equal to 800.
- (1,0) and (0,1) keep MP1 fully accepted in MTUs 1–2.
- After enforcing Business Rule 1, the admissible solutions are restricted to: (1,0) and (0,1)
- The configuration **(1,1)** is excluded because it would prevent the full acceptance of a higher-priced single-MTU bid.

Therefore, $(y_2, y_3) = (1,0)$ or $(0,1)$ and (0,0) is dominated.

The tie-breaking algorithm is applied since 2 solutions give the same total welfare. However, since both solutions have the exact same number of accepted block bids, the system will opt for the solution that has a lower ID number (let's assume it's the Block Bid of MP2).

6.6.4 Auction price computation

MTU	Total demand (MW)	ATC (MW)	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	30	25	0	0
10:15–10:30	30	25	0	0
10:30–10:45	20	25	0	0
10:45–11:00	20	25	0	0

Therefore, the auction price for each MTU is equal to 0.

6.6.5 Final auction results

MTU	ATC (MW)	MP1 (single-MTU bid) selected quantity (MW)	MP2 (block bid) selected quantity (MW)	MP3 (block bid) selected quantity (MW)	Auction Price (€/MWh)
10:00–10:15	25	10	10	0	10
10:15–10:30	25	10	10	0	10
10:30–10:45	25	-	10	0	0
10:45–11:00	25	-	10	0	0

6.7 Example 6 bis

MTU	ATC	MP1	MP2	MP3	MP4
10:00– 10:15	30	10 MW @ 20 €/MWh	Block 10 MW @ 10 €/MWh	Block 10 MW @ 10 €/MWh	10 MW @ 5 €/MWh
10:15– 10:30	30	10 MW @ 20 €/MWh			10 MW @ 5 €/MWh
10:30– 10:45	30	—			—
10:45– 11:00	30	—			—

6.7.1 Iteration 1: baseline scenario

The block bids are initially rejected. The ATC is fully available in each MTU for single-MTU bids.

Subproblem with $y_2=y_3=0$:

- $t=1$: accept 10 @20 and 10 @5 → welfare 250, ATC not fully used (capacity slack), $\lambda_1 = 0$.
- $t=2$: accept 10 @20 and 10 @5 → welfare 250, ATC not fully used (capacity slack), $\lambda_2 = 0$.
- $t=3$: → $\lambda_3 = 0$.
- $t=4$: → $\lambda_4 = 0$.

So:

- $V(0) = 500$.
- $\lambda = (0, 0, 0, 0)$.

Optimality cut from iteration 1:

$$\theta \leq 500$$

The first cut is simply: $\theta \leq 500$ (no dependence on y_2 and y_3).

Master moves to iteration 2.

6.7.2 Iteration 2: blocks accepted (1, 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y_2 + 400 \cdot y_3 + \theta)$$

Constraints:

4. $0 \leq \theta \leq 500$
5. $10y_2 + 10y_3 \leq 25$
6. $y \in \{0; 1\}$

To maximize, master sets $\theta=500$ and $y_2=y_3=1$, because there's no penalty yet. Objective:
 $400 \cdot 1 + 400 \cdot 1 + 500 = 1,300$.

Therefore, the master proposes (1,1).

Subproblem with $y_2=y_3=1$ and rem. ATC=10:

- t=1: accept 10 @20 → welfare 200, ATC used, $\lambda_1 = 20$.
- t=2: accept 10 @20 → welfare 200, ATC used, $\lambda_2 = 20$.
- t=3: → $\lambda_3 = 0$.
- t=4: → $\lambda_4 = 0$.

So:

- $V(1,1) = 400$.
- $\lambda = (20, 20, 0, 0)$.

Optimality cut from iteration 2:

$$\theta \leq 400 - 400(y_2 - 1) - 400(y_3 - 1)$$

$$\theta \leq 1200 - 400(y_2 + y_3)$$

$$\theta \leq 500 \text{ (optimality cut from iteration 1)}$$

Now the master is constrained by:

- $\theta \leq 500$.
- $\theta \leq 1200 - 400(y_2 + y_3)$.

(0,0):

- $\theta \leq 500$ and $\theta \leq 1200 - 0 = 1200 \rightarrow \theta \leq 500$
- $\text{Obj} = 0 + \theta \leq 500 \Rightarrow \text{best} = 500$.

(1,0) or (0,1): say (1,0):

- $\theta \leq 500$ and $\theta \leq 800 \Rightarrow \theta \leq 500$
- $\text{Obj} = 400 \cdot 1 + 400 \cdot 0 + \theta \leq 400 + 500 = 900 \Rightarrow \text{best} = 900$.

(1,1):

- $\theta \leq 500$ and $\theta \leq 400 \Rightarrow \theta \leq 400$
- $\text{Obj} = 400 \cdot 1 + 400 \cdot 1 + \theta \leq 800 + 400 = 1200$.

Master still prefers (1,1) with $\theta = 400 \Rightarrow$ objective 1,200

Benders selects $y^* = (1,1)$ (blocks accepted), $\theta^* = 400$, total welfare = $400 + 400 + 400 = 1200$

6.7.3 Applying business rules

If business rules 1, 2, 3 and 4 were to be taken into consideration by the Benders Decomposition:

6.7.3.1 Pattern A: $(y_2, y_3) = (0,0)$

No blocks accepted.

- Same as iteration 1:
 - $t=1$: MP1 10 MW @20 and MP4 10 MW @5 $\rightarrow 250$
 - $t=2$: MP1 10 MW @20 and MP4 10 MW @5 $\rightarrow 250$
 - $t=3,4$: 0.
- Total welfare: $W_{\{\text{blocks}\}}(0,0) + V(0,0) = 0 + 500 = 500$

6.7.3.2 Pattern B: $(y_2, y_3) = (1,0)$ or $(0,1)$ — exactly one block accepted

Remaining ATC per MTU = 20 MW

Subproblem, enforcing business rules:

- $t=1,2$:
 - Remaining ATC = 20.
 - MP1 wants 10 MW @20.
 - MP4 wants 10 MW @5.

- Welfare $t=1$ and $t=2$:
 - MP1 & MP4: $10 \times 20 + 10 \times 5 = 250$ each.
 - Block: $10 \times 10 = 100$ each.
 - Total per MTU = 350.
- $t=3,4$:
 - No single-MTU bids, but block uses 10 MW.
 - Welfare per MTU = block $10 \times 10 = 100$.

Total welfare = **900**.

Pattern (0,1) gives the same total welfare by symmetry.

6.7.3.3 Pattern C: $(y_2, y_3) = (1,1)$ — both blocks accepted

Now each block uses 10 MW per MTU, so total block usage per MTU = 20 MW.

Remaining ATC per MTU = 10 MW

Subproblem:

- $t=1,2$:
 - Remaining ATC = 10.
 - MP1 bid is 10 MW @20.
- $t=3,4$:
 - Still 10 MW residual ATC, but no single-MTU bids.

Configuration **(1,1)** is **feasible**.

Total welfare:

- MP2: $10 \times 10 \times 4 = 400$.
- MP3: $10 \times 10 \times 4 = 400$.
- MP1: $10 \times 10 \times 2 = 400$.

Total = **1200**.

Therefore, $(y_2, y_3) = (1,1)$.

6.7.4 Auction price computation

MTU	ATC usage (MW)	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	30	20	10
10:15–10:30	30	20	10
10:30–10:45	20 / 30	0	0
10:45–11:00	20 / 30	0	0

6.7.5 Final auction results

MTU	ATC (MW)	MP1 (single-MTU bid) selected quantity (MW)	MP2 (block bid) selected quantity (MW)	MP3 (block bid) selected quantity (MW)	MP4 (single-MTU bid) selected quantity (MW)	Auction Price (€/MWh)
10:00–10:15	30	10	10	10	0	10
10:15–10:30	30	10	10	10	0	10
10:30–10:45	30	-	10	10	-	0
10:45–11:00	30	-	10	10	-	0

6.8 Example 7

MTU	ATC	MP1 (single MTU)	MP2 (single MTU)	MP3 (single MTU)	MP4 (block)	MP5 (block)
10:00–10:15	100	30 MW @ 10 €/MWh	--	30 MW @ 10 €/MWh	Block: 55 MW @ 10 €/MWh	Block: 55 MW @ 10 €/MWh
10:15–10:30	100	40 MW @ 10 €/MWh	40 MW @ 10 €/MWh	--		
10:30–10:45	100	30 MW @ 10 €/MWh	10 MW @ 10 €/MWh	20 MW @ 10 €/MWh		
10:45–11:00	100	—	—	--		

6.8.1 Pre-processing

As part of pre-processing, on every MTU, single-MTU Bids of same price on the given MTU are merged into a bigger single-MTU Bid on the given MTU:

MTU	ATC	MP1 (single MTU)	MP2 (single MTU)	MP3 (single MTU)	MP4 (block)	MP5 (block)
10:00–10:15	100	Merged single-MTU Bid: 60 MW @ 10 €/MWh			Block: 55 MW @ 10 €/MWh	Block: 55 MW @ 10 €/MWh
10:15–10:30	100	Merged single-MTU Bid: 80 MW @ 10 €/MWh				
10:30–10:45	100	Merged single-MTU Bid: 70 MW @ 10 €/MWh				
10:45–11:00	100	—	—	--		

6.8.2 Benders decomposition algorithm

As an output of the Benders decomposition algorithm, CPLEX is allocating 55MW to only one block bid (either the block bid of MP4 or the block bid of MP5) and the remaining value of ATC (45MW) is allocated to every merged single-MTU Bid over the three first MTUs:

MTU	ATC	MP1 (single MTU)	MP2 (single MTU)	MP3 (single MTU)	MP4 (block)	MP5 (block)
10:00–10:15	100	Merged single-MTU Bid: 45 MW @ 10 €/MWh			Block: 0 MW @ 10 €/MWh	Block: 55 MW @ 10 €/MWh
10:15–10:30	100	Merged single-MTU Bid: 45 MW @ 10 €/MWh				
10:30–10:45	100	Merged single-MTU Bid: 45 MW @ 10 €/MWh				
10:45–11:00	100	—	—	--		

6.8.3 Auction price computation

MTU	AP (€/MWh)
10:00–10:15	10
10:15–10:30	10
10:30–10:45	10
10:45–11:00	10

6.8.4 Business rules validation

All business rules are satisfied since all bid prices are identical.

6.8.5 Tie-breaking algorithm

Two solutions are found by CPLEX with the same total welfare and the same number of selected block bids.

As an output of the tie-breaking algorithm, the block bid with the lowest bid ID is selected.

Let's assume this is the block bid of MP4:

MTU	ATC	MP1 (single MTU)	MP2 (single MTU)	MP3 (single MTU)	MP4 (block)	MP5 (block)
10:00–10:15	100	Merged single-MTU Bid: 45 MW @ 10 €/MWh			Block: 55 MW @ 10 €/MWh	Block: 0 MW @ 10 €/MWh
10:15–10:30	100	Merged single-MTU Bid: 45 MW @ 10 €/MWh				
10:30–10:45	100	Merged single-MTU Bid: 45 MW @ 10 €/MWh				
10:45–11:00	100	—	—	--		

6.8.6 Post processing: improved pro-rata

On every MTU, groups of single-MTU Bids with same price are split into individual single-MTU Bids.

On every MTU, and for every group of single-MTU Bids with the same price that is equal to the auction price on the given MTU (marginal Bids), the selected quantity of the marginal merged single-MTU Bid is redistributed to every individual single-MTU Bid that is part of this group of marginal Bids, according to the improved pro-rata algorithm.

Meaning that the improved pro-rata algorithm is applied only on single-MTU marginal Bids, it is not run on block bids.

MTU 10:00–10:15: an equal share (22.5MW) of the remaining ATC (45MW) is redistributed to the single-MTU Bid of MP1 and the single-MTU Bid of MP3. This equal share is rounded down to the nearest MW (22MW).

MTU 10:15–10:30: an equal share (22.5MW) of the remaining ATC (45MW) is redistributed to the single-MTU Bid of MP1 and the single-MTU Bid of MP2. This equal share is rounded down to the nearest MW (22MW).

MTU 10:30–10:45:

Iteration 1: on the first iteration of the improved pro-rata algorithm, an equal share (15MW) of the remaining ATC (45MW) is redistributed to the single-MTU Bid of MP1, the single-MTU Bid of MP2 and the single-MTU Bid of MP3. However, only 10MW is distributed to the single-MTU Bid of MP2 since the maximal bid quantity of the single-MTU Bid of MP2 is 10MW. The leftover of 5MW will be redistributed to the two other single-MTU Bids at the next iteration.

Iteration 2: an equal share (2.5MW) of the leftover (5MW) is redistributed to the single-MTU Bid of MP1 and the single-MTU Bid of MP3. The overall redistributed quantity to the single-MTU Bid of MP1 and the single-MTU Bid of MP3 is rounded down to the nearest MW (15 + 2.5 = 17.5MW is rounded down to 17MW).

6.8.7 Final auction results

MTU	ATC	MP1 (single MTU)	MP2 (single MTU)	MP3 (single MTU)	MP4 (block)	MP5 (block)
10:00–10:15	100	22 MW @ 10 €/MWh	--	22 MW @ 10 €/MWh	Block: 55 MW @ 10 €/MWh	Block: 0 MW @ 10 €/MWh
10:15–10:30	100	22 MW @ 10 €/MWh	22 MW @ 10 €/MWh	--		
10:30–10:45	100	17 MW @ 10 €/MWh	10 MW @ 10 €/MWh	17 MW @ 10 €/MWh		
10:45–11:00	100	—	—	--		