JAO eCAT

Report

Benders Decomposition Design

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1 Introduction

1.1 Context

The JAO capacity auctions allocate cross-border transmission capacity to market participants based on their bids. Historically, the auction clearing algorithm has operated **per hour** (Market Time Unit, MTU) and only with **single-hour bids**. Each hour is treated independently, and the algorithm selects the set of bids that maximizes **social welfare** under capacity constraints.

Market development and participant needs have led to the introduction of **block bids**, which allow participants to express “all-or-nothing” demand for capacity across multiple hours. At the same time, the time resolution of the auction has evolved: from a 60-minute resolution to a finer **15-minute MTU** resolution at corridor level. To accommodate both legacy 60-minute products and new 15-minute products **on the same border**, the following design choice is adopted:

- The **underlying MTU resolution** for the optimization is **15 minutes**.
- Every “60-minute bid” is modeled as a **block bid spanning the four 15-minute intervals** within the same clock hour.
- Block bids are indivisible and cannot be partially accepted, so it must be accepted in full or rejected in full (“all-or-nothing”).
- As a result, what we will call **single-hour bids** become, in implementation terms, **15-minute bids**, and **block bids** naturally represent:
    - 60-minute products (4 consecutive 15-minute MTUs within an hour), and
    - Longer multi-hour products (blocks spanning several 15-minute intervals across hours).

The inclusion of block bids significantly increases the complexity of the auction. This report explains how we integrate these block bids into the existing framework using **Benders decomposition**, in a way that:

- Maximizes overall welfare.
- Preserves consistency with existing hourly clearing.
- Respects business rules and fairness considerations.
- Remains computationally tractable for operational use.

This document explains the logic and implications of the chosen approach.

2 Current Auction Design: Single-Hour Bids

2.1 Objective: Maximizing Social Welfare

In the current setup (without block bids), the auction is cleared **separately for each hour**. For a given hour, the algorithm:

- Receives a set of **single-hour bids** (SHBs).
- Each bid has:
  - A **source** and **sink** control area.
  - A **quantity** (MW) requested.
  - A **bid price** (€/MWh).

The algorithm's goal is to:

maximizes social welfare through a basic optimization subject to available transmission capacity and bid quantity limits, followed by a post-processing logic for "pro-rata"s.

The objective function consists of the following:

$$obj = \max\left(\sum_{X,Y,B}\left[p_{b(x,y,b)}.\,d_{a(x,y,b)}\right]\right)$$

2.2 Constraints

For each hour, the main constraints are:

1. **Bid quantity constraints**
   - The accepted quantity for a bid cannot exceed the offered quantity.
   - Accepted quantity must be non-negative.
2. **Available Transfer Capacity (ATC) constraints**
   - The total net flow over each border cannot exceed the offered ATC for that hour.
   - The formulation may vary depending on the technical profile (TP), but conceptually:
     - "Total flow using that border ≤ ATC for that border and hour."

2.3 Auction price and Shadow prices

From this optimization, we derive **shadow prices** (dual variables) associated with ATC constraints. These shadow prices:

- Represent the **implicit marginal value of capacity** at a border and hour.
- When aggregated for all relevant constraints on a given source–sink path, they determine the **auction price (AP)** for that path.

Two simple rules then govern single-hour bids:

- Bids with price **above the AP**: fully accepted.
- Bids with price **below the AP**: fully rejected.

This gives a transparent, rule-based outcome in each hour and ensures that:

- Capacity is allocated to the highest-valued uses.
- The calculation of AP is consistent with the constraints and clearing outcome.

3 Introducing block bids

3.1 Business rules

Different business rules are possible. **They must be defined by the TSOs**.

To align block bids with the existing hourly pricing framework, we propose the following rules:

1. All **single-hour** bids with price **higher than AP** are fully accepted.
2. All **single-hour** bids with price **lower than AP** are fully rejected.
3. All **block bids** with block price **lower than the AP** (for at least one time interval) must be rejected.
4. All **block bids** with block price **higher than the AP** (for all time intervals) must be accepted.

3.2 Complexity induced by block bids

Block bids create two fundamental challenges:

1. **Inter-temporal linkage**
    - Decisions are no longer hour-by-hour: accepting a block ties together all hours in its block.
    - A block might be profitable in some hours and unprofitable in others, but still beneficial overall.
2. **All-or-nothing decisions**
    - Each block is represented by a binary decision: accepted (1) or rejected (0).

    - The problem becomes a **Mixed-Integer Linear Program (MILP)**, which is significantly harder to solve than a single LP per hour and cannot include the business rules logic in it.

Therefore, the **Benders decomposition** is used to retain the modular structure and leverage the existing hourly clearing logic.

4 Conceptual Framework: bilevel perspective

Before describing Benders, it is useful to think of the problem as a **bilevel model**:

- **Upper level (leader):** decides which block bids to accept (binary variables).
- **Lower level (follower):** performs the clearing of single-hour bids, for each MTU separately and simultaneously, given the remaining capacity after blocks are accepted.

The lower-level problem is the familiar:

- Per-hour welfare maximization for single-hour bids.
- Subject to ATC constraints and bid quantity constraints.

This lower-level problem is **convex (linear)** and satisfies **strong duality**, which means:

- The follower's optimal objective equals the optimal value of its dual problem.
- This property is a key enabler for Benders decomposition: it allows the use of dual (shadow price) information to guide the upper-level decisions.

5 Benders Decomposition

5.1.1 Basic principle

Benders decomposition splits the original complex problem into two linked pieces:

1. A **master problem**:
    - Handles the **complicating variables**: here, the binary block acceptance decisions.
    - Operates at a more aggregated level.
2. A **subproblem**:
    - Handles the **continuous variables**: here, the single-hour bids and flows.
    - Retains the existing structure of the hourly auction.

The method proceeds iteratively:

1. The master proposes a candidate set of block acceptances.
2. The subproblem clears the market for single-hour bids under those choices.
3. Dual information from the subproblem is used to add a **Benders cut** to the master problem, refining its understanding of the welfare impact.
4. The master is re-solved with the new cut, proposing a new combination of blocks.
5. This repeats until convergence.

At convergence, the combination of block acceptances and hourly clearing is **globally welfare-optimal** under the model and constraints.

5.1.2 Master problem (MILP, small)

The master problem is the "decision-maker" for block bids. It:

- Decides, for each block bid, whether it is **accepted (1)** or **rejected (0)**.
- Ensures that the sum of accepted block quantities on each border and hour does not exceed the available ATC, so that the subproblem remains feasible.
- Estimates the resulting welfare from the **single-hour bids**, without explicitly solving the full single-hour problem inside the master.

To achieve this, the master problem includes:

- Binary variables indicating **block acceptance**.
- A *proxy variable*, called **θ (theta)**, representing:
    - "The best total welfare contribution from **all** single-hour bids, given the capacity remaining after accepting these blocks."
- Feasibility cuts that enforce basic capacity consistency:
    - For each border and hour, the sum over all accepted blocks of (block quantity × acceptance binary) must be less than or equal to the ATC for that border and hour.

The **master objective** is:

$$F_{master} = \max\left(\sum_{bb}\sum_{h\in H_b} p_{bb}.d_{bb}.y_{bb} + \theta\right)$$

5.1.2.1 Why use θ?

θ is a **placeholder** for the subproblem's optimal welfare.

Initially, θ is unconstrained above (it could take any large value).

As we solve subproblems and collect information, we **add constraints (Benders cuts)** that lower the upper bound on θ, making it more accurate.

At the same time, the **feasibility cuts on block quantities vs. ATC** ensure that the master never proposes a block-acceptance pattern that would make the subproblem infeasible.

This feasibility cut in the master problem ensures that the sum of accepted block quantities never exceeds ATC on any border and hour. Because the subproblem allows zero accepted quantity of single-hour bids and does not include additional minimum-acceptance or intertemporal constraints, this is sufficient to guarantee that the subproblem is always feasible.

This keeps the master problem:

- **Relatively small**: mostly binary block variables plus θ.
- **Efficient and robust**: we avoid wasted iterations on subproblems that would fail due to impossible capacity over-commitment by blocks.

The master problem is structured to answer: "Which blocks should be accepted?" in a compact and feasible way, while treating the detailed hourly clearing of single-hour bids through θ and Benders cuts. This separation makes the problem solvable at scale and allows adding business-specific "cuts" to deal with paradoxical block bids. Feasibility cuts ensure that the sum of accepted block quantities never exceeds ATC on any border and hour, so the combined master + subproblem formulation remains feasible by construction.

5.1.3 Subproblem (LP, fast)

For a fixed block acceptance pattern (a fixed $y = \bar{y}$) and for each MTU:

- The subproblem receives the **remaining ATC per border and hour**, after accounting for capacity used by accepted blocks.
- It then runs the **standard single-hour auction LP**, for all hours separately:
    - Maximizes welfare of single-hour bids.
    - Respects ATC and per-bid quantity constraints.

Its outputs include:

- The **optimal welfare contribution** from all single-hour bids under the given block pattern.

- The **dual variables (shadow prices)** associated with the ATC and bid constraints.

The subproblem objective is the following:

$$F_{sub} = V(\bar{y}) = \max\left(\sum_{b} p_{b,h}.d_{b,h}\right)$$

The problem is a standard single hour auction clearing LP. It is solved for each MTU separately and for all MTUs simultaneously.

5.1.3.1 Strong duality and shadow prices

The subproblem is a **linear program** with convex structure, so:

- It satisfies **strong duality**:
    - The primal objective (max welfare) equals the dual objective at optimum.
- The dual variables (shadow prices):
    - Reflect the marginal value of capacity on each border and hour.
    - Are used to build **Benders cuts** for the master problem.

The subproblem precisely answers: "Given this set of accepted blocks, how can we best clear single-hour bids and what is the value of remaining capacity?" The associated shadow prices then tell the master how "expensive" block capacity usage is, so it can adjust its block decisions intelligently.

5.1.4 Benders Optimality Cut

After solving the subproblem for a given pattern of block acceptances:

- We know:
    - The actual welfare from single-hour bids, given these blocks.
    - The shadow prices for ATC constraints.

From this, we construct a **Benders cut**: a linear inequality (Appendix A) that relates:

- The **block decisions** (which blocks are accepted).
- The **maximum welfare** that the subproblem can deliver (represented by θ).

This cut says:

"If you accept these blocks (and in general, blocks that use capacity in certain expensive hours), the best welfare you can get from single-hour bids is no more than a certain value."

Each cut provides a tighter upper bound on θ, helping the master problem approximate the true welfare impact of block decisions.

The key economic signals come from **shadow prices**:

- When capacity for a given hour and border is very valuable (high shadow price), any block that uses this capacity is "expensive" to the system.
- The Benders cut reflects this by giving such blocks a more negative impact on θ whenever they consume capacity in those hours.

Thus:

- Blocks that consume capacity in **peak-value hours** get penalized more.
- Blocks that mainly use **low-value or uncongested hours** are less penalized.

Over successive iterations:

- The master learns that some blocks are systematically detrimental to welfare (due to high shadow prices at the times they use capacity) and tends to reject them.
- Other blocks that harmonize well with high-value single-hour bids, or that fill low-value capacity, are more likely to be accepted.

θ shall be lower bounded either by 0 or by a certain value.

5.1.5 Iterative Process and Convergence

The Benders decomposition follows this loop:

1. **Solve initial master**
    - Start with no Benders cuts.
    - Include feasibility cuts that enforce:
        - For each border and hour, the sum of (block quantity × acceptance binary) ≤ ATC.
    - θ is unconstrained above, so the master will initially tend to accept blocks that look valuable on their own (based on block price and quantity), since it "assumes" the single-hour welfare can still be high.
    - However, one can start from a neutral block acceptance which will provide the baseline welfare which is the maximum welfare one can get from the single-hour

bids without any block bids. Moreover, its duals (shadow prices) provide meaningful sensitivities. When no blocks are accepted, the shadow prices reflect the “pure” congestion value per hour. So, if an hour is tight (high shadow price), then the next iteration’s cut will penalize accepting blocks that consume capacity there. Also, it’s a conservative initialization that will measure how much value single-hour bids produce, then progressively test block inclusion through cuts.

- Obtain:
  - Candidate block acceptance pattern y (which blocks are tentatively accepted).
  - Current θ value (estimated welfare from single hour bids).

  Because of the feasibility cuts, any “y” chosen by the master already respects basic ATC limits across all block hours. This guarantees that the subproblem can find a feasible dispatch for single hour bids (possibly with zero accepted quantity if capacity is fully used by blocks).

2. **Solve subproblem**
   - Fix y from the master.
   - Adjust ATC per hour and border based on accepted blocks (remaining capacity = ATC – block usage).
   - Solve the single-hour LP problem to optimality across all hours.
   - Obtain:
     - Actual welfare from single hour bids.
     - Dual variables (shadow prices) for ATC constraints.
3. **Compute Auction Price**
   - The Auction price is computed by choosing the cheapest accepted bid (regular bid or block bid).
   - This unifies the definition of the auction price across the master problem and the subproblem, since we cannot rely on the duals of the subproblem anymore.
4. **Generate Cut**
   - Use strong duality and shadow prices to derive a Benders optimality cut.
   - The cut limits θ as a function of block decisions and capacity usage.
5. **Add paradoxical block cuts**

- If, based on realized prices and the clearing result, a block is identified as paradoxically accepted (or fails a specific business rule):
    - Add a constraint forcing that block to be rejected in subsequent iterations (e.g., y_block = 0).

6. **Update Master and re-solve**
    - Add the new cut to the master problem.
    - Any additional cuts from paradoxical block handling or other business rules.
    - Feasibility cuts on block quantities vs. ATC remain in place throughout the process.
    - Return to Step 1.
    - Re-solve the master to obtain a new set of block decisions and a refined θ.
7. **Repeat** until:
    - For the chosen block pattern, the θ in the master equals the true welfare from the subproblem.
    - No new cuts further improve the solution.
    - At this point, we have convergence.

5.1.6 Guaranteed maximum welfare

Because each Benders cut is derived from exact dual information of a convex LP:

- The master's upper bound on welfare becomes progressively tighter.
- The final solution is **globally optimal**:
    - There is no other combination of block acceptances and single-hour dispatch that could yield higher total welfare under the modeled constraints.

In other words:

> *The iterative process systematically eliminates overly optimistic views of single-hour welfare for each block combination, until it finds the combination for which expectations and reality match. That combination is the welfare-maximizing solution.*

5.1.7 Fairness and non-Discrimination

5.1.7.1 Equal treatment in Welfare Objective

Both single-hour and block bids are evaluated in the same welfare framework:

- Each bid's contribution is:
  - Quantity × price.
- The optimization allocates capacity to the combination of bids that gives the highest total welfare.

This ensures that:

- There is no explicit preference for one bid type over the other in the objective.
- Every euro of surplus is treated equally, regardless of its source.

5.1.7.2 Shadow Prices and Non-Discriminatory Capacity Valuation

Shadow prices (dual variables) provide a **uniform valuation of capacity**:

- All bids (block or single-hour) face the **same shadow price** for using capacity on a given border and hour.
- If capacity is scarce and valuable for a given period, both block and single-hour bids are "charged" the same opportunity cost.

Thus:

- The optimization model does not discriminate between bid types in valuing capacity.
- Differences in outcomes stem from the structural differences:
  - Single-hour bids are flexible and can be accepted or rejected hour by hour.
  - Block bids must be accepted or rejected in full, which can create paradoxical situations.

5.1.8 Block vs Single-Hour Bids: Are We Favouring One?

5.1.8.1 Methodological neutrality

From a model design perspective:

- The welfare objective treats all bids symmetrically.
- Capacity constraints, shadow prices, and the rules around AP apply consistently.

However, the **structural properties** of the bids imply differences:

- **Single-hour bids**:
  - Have a simple and local rule relative to AP.
  - Each hour is independent; a bid can be accepted in one hour and rejected in another, even if from the same participant.

- **Block bids**:
    - Must be “all-or-nothing” across multiple hours.
    - May be rejected even if their price looks attractive in most individual hours (paradoxically rejected).
    - May be accepted even if some hours in the block are out of the money, as long as the **overall** impact is positive.

5.1.8.2 Perception of favouritism

In practice, this can lead to perceptions such as:

- “Block bids are harder to get accepted, even with a competitive price.”
- “Single-hour bids always follow the simple AP rule, but blocks don’t.”

These differences are **not caused by Benders decomposition itself**, but by:

- The **all-or-nothing nature** of blocks.
- The requirement to maximize total welfare over multiple hours.

Therefore:

> *The algorithm does not intentionally favour one type of bid over another. It systematically chooses the combination of single-hour and block bids that maximizes total welfare, but block bids are more constrained by design and hence more prone to paradoxical acceptance/rejection.*

5.1.9 Handling the paradoxically accepted/rejected block bids

5.1.9.1 Paradoxically rejected blocks

A **paradoxically rejected** block is:

- A block that appears to be “in the money” based on observed auction prices but is rejected.
- This can occur because:
    - Accepting it would crowd out higher-value single-hour bids or other blocks in one or more hours.
    - The welfare gain in some hours is outweighed by welfare losses in others.

Paradoxical rejections are an inherent feature of block products under welfare maximization with capacity constraints.

5.1.9.2 Paradoxically accepted blocks

A **paradoxically accepted** block would be:

- A block that, under the applied market prices, would not be profitable for the participant, yet is accepted.
- To avoid forcing participants into loss-making positions, business rule 3 (in Chapter 3.2) is enforced.

5.1.9.3 How does Benders Decomposition handle the paradoxical blocks

Benders decomposition alone does not resolve issues of paradoxical acceptance or rejection. It:

- Efficiently finds the welfare-maximizing solution under the given constraints.
- But does not, by itself, enforce revenue adequacy or "no loss" conditions for individual blocks.

Only paradoxically accepted blocks are going to be ruled out in the Benders Decomposition approach through business rule 3. To address these blocks **an integrated constraints approach** is followed. It consists of adding explicit constraints to the optimization that:

  - Prevent paradoxical acceptances by bounding block revenue relative to prices.
  - If a block is found paradoxical in an iteration, add a cut stating that this block must not be accepted.

5.1.10 Role of the business rules in Benders Decomposition

The business rules described in this report are not a technical necessity of Benders decomposition; they are a **deliberate market design choice**. Their purpose is to shape the welfare-maximizing outcome so that it aligns with JAO's market principles and participants' expectations.

In a purely mathematical sense, Benders decomposition would simply:

- Decide block acceptances in the master problem,
- Maximize welfare from single-hour bids in the subproblem, and
- Iterate until overall welfare is maximized, without regard to how individual bids fare under the resulting prices.

What distinguishes our implementation is that we explicitly **embed business rules into the Benders loop**:

- Single-hour rules (“bids above AP fully accepted, bids below AP fully rejected”) are enforced in the subproblem.
- Block rule 3 (“block price must not be lower than the AP over its hours”) is checked at each iteration; any pattern that leads to a paradoxically accepted block is ruled out by adding a cut (e.g. forcing ($y_b = 0$) for that block).
- Additional business-oriented criteria, such as tie-breaking rules between welfare-equivalent block patterns, are implemented at the master level (e.g. preferring solutions with more single-hour welfare or fewer accepted blocks).

Benders decomposition is the enabling technical framework: it keeps the problem solvable at scale and exposes the right dual information (shadow prices) to build welfare-based cuts. The **business rules are what give this framework its market shape**. They allow to “manipulate” the optimization in a controlled and transparent way:

- **Maximize total welfare** under physical and economic constraints, and at the same time
- **Enforce market rules** such as no-loss conditions for blocks, priority patterns for single-hour bids, and consistent pricing logic.

5.1.11 Tie-breaking rules

In some configurations, the welfare-maximizing problem admits **multiple block-acceptance patterns with the same total welfare**. This is already visible in simple one-block examples:

- Example 1:
    - Rejecting the block ($y = 0$) and fully clearing the single-hour bids yields total welfare = 400.
    - Accepting the block ($y = 1$) and crowding out the single-hour bids also yields total welfare = 400.
- Example 4:
    - Rejecting the block ($y = 0$) and accepting the single-hour bids in the first two MTUs yields total welfare = 400.
    - Accepting the block ($y = 1$) and crowding out the single-hour bids yields the same 400.

From a pure welfare perspective, both patterns are equally optimal. However, in an operational implementation this ambiguity must be resolved in a **systematic and transparent way**, so that the algorithm always produces a unique outcome.

We therefore introduce **explicit tie-breaking rules** at the master-problem level, to select one block-acceptance pattern among welfare-equivalent alternatives.

5.1.11.1 Design choice

Different tie-breaking philosophies are possible. **They must be defined by the TSOs**. Two examples can be found below:

1. **Preference for single-hour bids (SH-first):**
   Among all welfare-maximizing solutions, prefer those that keep as much welfare as possible in single-hour bids. Intuitively, blocks are used only when they clearly bring additional welfare beyond what the single-hour bids can provide.

2. **Preference for fewer accepted blocks (minimal block usage):**
   Among all welfare-maximizing solutions, prefer solutions with the smallest number of accepted blocks. This keeps the outcome closer to the legacy single-hour world and reduces the number of all-or-nothing commitments.

Both philosophies lead to the same choice in Examples 1 and 4 (rejecting the block), but they are conceptually distinct and could differ in more complex situations.

5.1.11.2 Implementation

We implement tie-breaking as a lexicographic optimization on top of the Benders master problem:

Once Benders has converged and the maximum welfare W* is known, we **re-solve the master problem** with an additional constraint that fixes the welfare at its optimum:

$$\sum_{b \in B} W_b y_b + \theta \ \geq \ W^*$$

and a new secondary objective, depending on the chosen philosophy:

SH-first tie-breaker (prefer single-hour bids):

$$\max(\theta)$$

Among all block patterns that yield total welfare W*, we pick the one that maximizes θ, i.e. that preserves the largest possible welfare in single-hour bids.

Minimal-blocks tie-breaker (prefer fewer blocks):

$$\min\left(\sum_{b} y_b\right)$$

Among all welfare-maximizing block patterns, we pick the one with the smallest number of accepted blocks.

In practice, this is implemented as a **second pass** on the master problem after the standard Benders loop has converged. No new subproblem solves are needed: all Benders cuts are kept; only the objective and one additional constraint are changed for the tie-breaking pass.

6 Examples

This section contains examples of corner cases to see how Benders Decomposition will tackle them:

6.1 Example 1

Time	ATC	MP1 (single-hour bid)	MP2 (block bid)
10:00–10:15	10	10 MW @ 15 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	10 MW @ 10 €/MWh	
10:30–10:45	10	10 MW @ 10 €/MWh	
10:45–11:00	10	10 MW @ 5 €/MWh	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

6.1.1 Iteration 1: baseline scenario

If we decide to initialize the Benders decomposition to y=0, the block bid is initially rejected. The ATC is fully available in each MTU for single-hour bids.

Subproblem with y=0:

- t=1: accept 10 @15 → welfare 150, ATC used, λ_1 = 15.
- t=2: accept 10 @10 → welfare 100, ATC used, λ_2 = 10.
- t=3: accept 10 @10 → welfare 100, λ_3 = 10.
- t=4: accept 10 @5 → welfare 50, λ_4 = 5.

So:

- V(0) = 150 + 100 + 100 + 50 = 400.

- λ = (15, 10, 10, 5).

Optimality cut from iteration 1:

$$\theta \leq 400 - 400\,y$$

Master moves to iteration 2 and accepts the block bid.

6.1.2 Iteration 2: block bid accepted (y = 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $\theta \leq 400 - 400\,y$
2. $10y \leq 10$
3. $y \in \{0;1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with y = 1:
    - The remaining capacity for single-hour bids = 0 MW in all 4 MTUs
    - No capacity left, so no MP1 bids can be accepted.
    - Single-hour welfare: V(1) = 0.
    - λ = (0, 0, 0, 0).

Optimality cut from iteration 2:

$$\theta \leq 0 - 400\,(y - 1)$$

$$\theta \leq 400 - 400\,y$$

We substitute the cut:

- For y=0: θ ≤ 400 → best θ is 400 → objective = 0·400 + 400 = 400.
- For y=1: θ ≤ 400(1−1) = 0 → best θ is 0 → objective = 400·1 + 0 = 400.

Converged solution: any y ∈ {0,1}, total welfare 400.

6.1.3 Auction price computation

If block bid is rejected:

MTU	Shadow price	AP (€/MWh)
10:00–10:15	15	**15**
10:15–10:30	10	**10**
10:30–10:45	10	**10**
10:45–11:00	5	**5**

If block bid is accepted:

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	0	**10**
10:15–10:30	0	**10**
10:30–10:45	0	**10**
10:45–11:00	0	**10**

6.1.4 Results interpretation

Welfare is identical in both cases, so the Benders master problem has at least two optimal solutions (y = 0 or y = 1) from a pure welfare perspective.

Detailed Benders interpretation

- Subproblem with y=1 gives actual welfare from single-hour bids = 0 and duals on ATC indicate that marginal value of capacity would be around 15, 10, 10, 5 €/MWh in the corresponding hours if capacity were available.
- Benders cut basically says:
  "If you allocate all capacity to this block, the best welfare from single-hour bids is 0 and the opportunity cost is exactly equal to the value you would have obtained with single-hour bids."

- Therefore, total welfare is 400 either way.
- In practice the implementation would pick one (depending on tie breaking algorithm put in place).

6.2 Example 2

Time	ATC	MP1	MP2 (block)
10:00–10:15	10	10 MW @ 14 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	10 MW @ 10 €/MWh	
10:30–10:45	10	10 MW @ 10 €/MWh	
10:45–11:00	10	10 MW @ 5 €/MWh	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

- Feasibility cut (block_quantity <= ATC): $10y \leq 10$
- $y \in \{0; 1\}$

6.2.1 Iteration 1: baseline scenario

The block bid is initially rejected. The ATC is fully available in each MTU for single-hour bids.

Subproblem with y=0:

- t=1: accept 10 @15 → welfare 150, ATC used, λ_1 = 14.
- t=2: accept 10 @10 → welfare 100, ATC used, λ_2 = 10.
- t=3: accept 10 @10 → welfare 100, λ_3 = 10.
- t=4: accept 10 @5 → welfare 50, λ_4 = 5.

So:

- V(0) = 140 + 100 + 100 + 50 = 390.
- λ = (14, 10, 10, 5).

Optimality cut from iteration 1:

$$\theta \leq \ 390 \ - \ 390 \, y$$

Master moves to iteration 2 and accepts the block bid.

6.2.2 Iteration 2: block accepted (y = 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $\theta \leq \ 390 \ - \ 390 \, y$
2. $10y \leq \ 10$
3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with y = 1:
    - The remaining capacity for single-hour bids = 0 MW in all 4 MTUs
    - No capacity left, so no MP1 bids can be accepted.
    - Single-hour welfare: V(1) = 0.
    - λ = (0, 0, 0, 0).

Optimality cut from iteration 2:

$$\theta \leq \ 0 - \ 390 \, (y - 1)$$

$$\theta \leq \ 390 - \ 390 \, y$$

Objective:

- y=0 ⟶ 400·0 + θ ≤ 390 = 390.
- y=1 ⟶ 400·1 + θ ≤ 400 + 0 = 400.

Therefore, the master chooses to accept the block bid. Benders converges (one iteration is enough) to:

y* = 1 (block accepted), θ* = 0, total welfare = 400

6.2.3 Auction price computation

The auction price is equal to the cheapest accepted bid which in this case is the block bid.

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	0	**10**
10:15–10:30	0	**10**
10:30–10:45	0	**10**
10:45–11:00	0	**10**

6.2.4 Results interpretation

Welfare is higher when the block is accepted:

- Welfare (y=1) = 400
- Welfare (y=0) = 390
  → block **increases welfare** by 10 compared to only single-hour bids.

From the Benders decomposition angle:

- First iteration: Subproblem with y=0 yields welfare 390; dual prices show that capacity is especially valuable in the first interval (14 €/MWh).
- Evaluating y=1, subproblem gives 0 welfare from single-hour bids: total = 400 via block.
- Master receives a cut indicating that using capacity for the block is beneficial — θ under "block accepted" can be higher in combination with the block's own contribution.
- Final master optimum: **accept block** (y = 1), since 400 > 390.

6.3 Example 3

Time	ATC	MP1 (single-hour)	MP2 (block)
10:00–10:15	10	10 MW @ 15 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	— (no bid)	
10:30–10:45	10	— (no bid)	
10:45–11:00	10	— (no bid)	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

- Feasibility cut (block_quantity <= ATC): $10y \leq 10$
- $y \in \{0; 1\}$

6.3.1 Iteration 1: baseline scenario

The block bid is initially rejected. The ATC is fully available in each MTU for single-hour bids.

Subproblem with y=0:

- t=1: accept 10 @15 → welfare 150, ATC used, λ_1 = 15.
- t=2: → λ_2 = 0.
- t=3: → λ_3 = 0.
- t=4: → λ_4 = 0.

So:

- V(0) = 150.
- λ = (15, 0, 0, 0).

Optimality cut from iteration 1:

$$\theta \leq 150 - 150\, y$$

Master moves to iteration 2 and accepts the block bid.

6.3.2 Iteration 2: block accepted (y = 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $\theta \leq 150 - 150\, y$
2. $10y \leq 10$

3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with y = 1:
    - The remaining capacity for single-hour bids = 0 MW in all 4 MTUs
    - No capacity left, so no MP1 bids can be accepted.
    - Single-hour welfare: V(1) = 0.
    - λ = (0, 0, 0, 0).

Optimality cut from iteration 2:

$$\theta \leq 0 - 150\,(y - 1)$$

$$\theta \leq 150 - 150\,y$$

We substitute the cut:

- For y=0: θ ≤ 150 → best θ is 150 → objective = 0·150 + 150 = 150.
- For y=1: θ ≤ 150(1−1) = 0 → best θ is 0 → objective = 400·1 + 0 = 400.

Benders selects y*=1 (block accepted), θ*=0, total welfare=400.

6.3.3 Auction price computation

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	0	**10**
10:15–10:30	0	**10**
10:30–10:45	0	**10**
10:45–11:00	0	**10**

6.3.4 Results interpretation

Welfare is higher when the block is accepted:

- Welfare (y=1) = 400

- Welfare (y=0) = 150
  → block **massively increases welfare** compared to only single-hour bids.

From a Benders Decomposition perspective:

- Master tries y=0 first.
- Subproblem yields θ=150, with strong dual in first interval and slack in others.
- Master then tries y=1:
  - Subproblem welfare from single-hour bids = 0.
  - But adding block's own welfare (400) yields total of 400.
- The Benders cut makes it clear that capacity in the three intervals with no MP1 bids is essentially free capacity; using it for a block adds pure welfare with no opportunity cost.
- The regular bid in the first MTU is crowded out by the block because the block's additional welfare in the other 3 MTUs more than compensates the forgone 150.
- Therefore, the master ultimately chooses y=1.

6.4 Example 4

Time	ATC	MP1 (single-hour)	MP2 (block)
10:00–10:15	10	10 MW @ 20 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	10 MW @ 20 €/MWh	
10:30–10:45	10	—	
10:45–11:00	10	—	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

- Feasibility cut (block_quantity <= ATC): $10y \leq 10$
- $y \in \{0; 1\}$

6.4.1 Iteration 1: baseline scenario

The block bid is initially rejected. The ATC is fully available in each MTU for single-hour bids.

Subproblem with y=0:

- t=1: accept 10 @20 → welfare 200, ATC used, $\lambda_1 = 20$.
- t=2: accept 10 @20 → welfare 200, ATC used, $\lambda_2 = 20$.
- t=3: → $\lambda_3 = 0$.
- t=4: → $\lambda_4 = 0$.

So:

- V(0) = 400.
- λ = (20, 20, 0, 0).

Optimality cut from iteration 1:

$$\theta \leq 400 - 400\, y$$

Master moves to iteration 2 and accepts the block bid.

6.4.2 Iteration 2: block accepted (y = 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $\theta \leq 400 - 400\, y$
2. $10y \leq 10$
3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with y = 1:
  - The remaining capacity for single-hour bids = 0 MW in all 4 MTUs
  - No capacity left, so no MP1 bids can be accepted.
  - Single-hour welfare: V(1) = 0.
  - λ = (0, 0, 0, 0).

Optimality cut from iteration 2:

$$\theta \leq 0 - 400\,(y-1)$$

$$\theta \leq 400 - 400\,y$$

We substitute the cut:

- For y=0: θ ≤ 400 → best θ is 400 → objective = 0·400 + 400 = 400.
- For y=1: θ ≤ 150(1−1) = 0 → best θ is 0 → objective = 400·1 + 0 = 400.

Converged solution: any y ∈ {0,1}, total welfare 400. Tie breaking rule will choose which solution to go for.

6.4.3 Auction price computation

If block bid is rejected:

MTU	**Shadow price (€/MWh)**	**AP (€/MWh)**
10:00–10:15	20	**20**
10:15–10:30	20	**20**
10:30–10:45	0	**0**
10:45–11:00	0	**0**

If block bid is accepted:

MTU	**Shadow price (€/MWh)**	**AP (€/MWh)**
10:00–10:15	**0**	**10**
10:15–10:30	**0**	**10**
10:30–10:45	**0**	**10**
10:45–11:00	**0**	**10**

6.4.4 Results interpretation

Welfare is identical in both cases, so the Benders master problem has at least two optimal solutions (y = 0 or y = 1) from a pure welfare perspective.

Detailed Benders interpretation

- Subproblem with y=1 gives actual welfare from single-hour bids = 0 and duals on ATC indicate that marginal value of capacity would be around 15, 10, 10, 5 €/MWh in the corresponding hours if capacity were available.
- Benders cut basically says:
  "If you allocate all capacity to this block, the best welfare from single-hour bids is 0 and the opportunity cost is exactly equal to the value you would have obtained with single-hour bids."
- Therefore, total welfare is 400 either way.
- In practice the implementation would pick one (depending on tie breaking algorithm put in place).

6.5 Example 5

Time	ATC	MP1 (single-hour)	MP2 (block)
10:00–10:15	10	2 MW @ 20 €/MWh	Block: 10 MW @ 10 €/MWh
10:15–10:30	10	2 MW @ 20 €/MWh	
10:30–10:45	10	—	
10:45–11:00	10	—	

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

- Feasibility cut (block_quantity <= ATC): $10y \leq 10$
- $y \in \{0; 1\}$

6.5.1 Iteration 1: baseline scenario

The block bid is initially rejected. The ATC is fully available in each MTU for single-hour bids.

Subproblem with y=0:

- t=1: accept 2 @20 → welfare 40, ATC used, $\lambda_1 = 20$.
- t=2: accept 2 @20 → welfare 40, ATC used, $\lambda_2 = 20$.
- t=3: → $\lambda_3 = 0$.
- t=4: → $\lambda_4 = 0$.

So:

- V(0) = 80.
- λ = (20, 20, 0, 0).

Optimality cut from iteration 1:

$$\theta \leq 80 - 400\,y$$

Master moves to iteration 2 and accepts the block bid.

6.5.2 Iteration 2: block accepted (y = 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y + \theta)$$

Constraints:

1. $0 \leq \theta \leq 80 - 400\,y$
2. $10y \leq 10$
3. $y \in \{0; 1\}$

The block uses 10 MW in **all** 4 intervals. With the ATC being equal to 10 MW, so:

- Subproblem with y = 1:
    - The remaining capacity for single-hour bids = 0 MW in all 4 MTUs
    - No capacity left, so no MP1 bids can be accepted.
    - Single-hour welfare: V(1) = 0.
    - λ = (0, 0, 0, 0).

Optimality cut from iteration 2:

$$\theta \leq 400 - 400\,y$$

$\theta \leq 80 - 400\,y$ (optimality cut generated in iteration 1)

We substitute the cut:

- For y=0: θ ≤ min(400; 80) → best θ is 80 → objective = 0·400 + 80 = 80.
- For y=1: θ≤ min(0; -320)= -320 however theta should be non-negative so this solution cannot be accepted.

Benders cannot select y*=1 because theta is becoming negative so the block is rejected.

6.5.3 Auction price computation

MTU	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	20	**20**
10:15–10:30	20	**20**
10:30–10:45	0	**0**
10:45–11:00	0	**0**

6.6 Example 6

Time	ATC	MP1	MP2	MP3
10:00–10:15	25	10 MW @ 20 €/MWh	Block 10 MW @ 10 €/MWh	Block 10 MW @ 10 €/MWh
10:15–10:30	25	10 MW @ 20 €/MWh		
10:30–10:45	25	—		
10:45–11:00	25	—		

The master problem in Benders Decomposition is defined as follows:

$$Master = \max(400 \cdot y_2 + 400 \cdot y_3 + \theta)$$

Constraints:

- Feasibility cut (block_quantity <= ATC): $10y_2 + 10y_3 \leq 25$
- $y_2, y_3 \in \{0; 1\}$

Feasibility (capacity):

$10y_2 + 10y_3 \leq 25$ in each MTU; so the constraint in master is $10y_2+10y_3 \leq 25$ (MTUs 3–4) and $10y_2+10y_3$ +10 (MP1) ≤ 25 in MTUs 1–2, but **MP1 is not in the master**, so the master only imposes:

- $10y_2 + 10y_3 \leq 25$ (blocks alone not exceeding ATC).

Hence $(y_2,y_3)=(1,1)$ is *feasible* in master; the capacity conflict with MP1 is discovered via duals in the subproblem.

6.6.1 Iteration 1: baseline scenario

The block bids are initially rejected. The ATC is fully available in each MTU for single-hour bids.

Subproblem with y_2=y_3=0:

- t=1: accept 10 @20 → welfare 200, ATC not fully used (capacity slack), $\lambda_1 = 0$.
- t=2: accept 10 @20 → welfare 200, ATC not fully used (capacity slack), $\lambda_2 = 0$.
- t=3: → $\lambda_3 = 0$.
- t=4: → $\lambda_4 = 0$.

So:

- V(0) = 400.
- λ = (0, 0, 0, 0).

Optimality cut from iteration 1:

$$\theta \leq 400$$

The first cut is simply: $\theta \leq 400$ (no dependence on y_2 and y_3).

Master moves to iteration 2.

6.6.2 Iteration 2: blocks accepted (1, 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y_2 + 400 \cdot y_3 + \theta)$$

Constraints:

1. $0 \leq \theta \leq 400$
2. $10y_2 + 10y_3 \leq 25$
3. $y \in \{0; 1\}$

To maximize, master sets θ=400 and y_2=y_3=1, because there's no penalty yet. Objective:

400·1 + 400·1 + 400 = 1,200.

Therefore, the master proposes (1,1).

Subproblem with y_2=y_3=1 and rem. ATC=5:

- t=1: accept 5 @20 → welfare 100, ATC used, λ_1 = 20.
- t=2: accept 5 @20 → welfare 100, ATC used, λ_2 = 20.
- t=3: → λ_3 = 0.
- t=4: → λ_4 = 0.

So:

- V(1,1) = 200.
- λ = (20, 20, 0, 0).

Optimality cut from iteration 2:

$$\theta \leq 200 - 400(y_2 - 1) - 400(y_3 - 1)$$

$$\theta \leq 1000 - 400(y_2 + y_3)$$

Now the master is constrained by:

- θ ≤ 400.
- θ ≤ 1000 − 400(y_2 + y_3).

(0,0):

  - θ ≤400 and θ ≤1000−0=1000 → θ≤400
  - Obj = 0 + θ ≤ 400 ⇒ best=400.

(1,0) or (0,1): say (1,0):

- $\theta \leq 400$ and $\theta \leq 600 \Rightarrow \theta \leq 400$
- Obj=400·1 +400·0 + θ ≤400 +400 =800 ⇒ best=800.

(1,1):

- $\theta \leq 400$ and $\theta \leq 200 \Rightarrow \theta \leq 200$
- Obj=400·1+400·1+θ ≤800+200=1000.

Master still prefers (1,1) with θ=200 ⇒ objective 1,000

Benders selects y*=(1,1) (blocks accepted), θ*=200, total welfare=400 + 400 + 200 = 1000

6.6.3 Introducing business rules

If business rules 1, 2, 3 and 4 were to be taken into consideration by the Benders Decomposition:

6.6.3.1 Pattern A: (y_2, y_3) = (0,0)

No blocks accepted.

- Same as iteration 1:
  - t=1: MP1 10 MW @20 → 200.
  - t=2: MP1 10 MW @20 → 200.
  - t=3,4: 0.
- Total welfare: W_{blocks}(0,0) + V0,0) = 0 + 400 = 400

6.6.3.2 Pattern B: (y_2, y_3) = (1,0) or (0,1) — exactly one block accepted

Remaining ATC per MTU = 15 MW

Subproblem, enforcing business rules:

- t=1,2:
  - Remaining ATC = 15.
  - MP1 wants 10 MW @20.
  - Capacity ≥ 10, so MP1 must be fully accepted at 10 MW.
  - No other regular bids, so:
    - MP1 =10 MW, block MP2=10 MW (fixed), total 20 ≤25.
  - Welfare t=1 and t=2:

        - MP1: 10×20 = 200 each.
        - Block: 10×10 = 100 each.
        - Total per MTU = 300.
- t=3,4:
    - No regular bids, but block uses 10 MW.
    - Welfare per MTU = block 10×10 = 100.

Total welfare = **800**.

Pattern (0,1) gives the same total welfare by symmetry.

6.6.3.3 Pattern C: (y_2, y_3) = (1,1) — both blocks accepted

Now each block uses 10 MW per MTU, so total block usage per MTU = 20 MW.

Remaining ATC per MTU = 5 MW

Subproblem:

- t=1,2:
    - Remaining ATC = 5.
    - MP1 bid is 10 MW @20.
    - Business rule: **MP1 must be fully accepted** because its bid price is higher than the auction Price (AP = 10).
    - But here capacity for the regular bid is only 5 MW, **not enough** to accept 10 MW.
    - That means we **must** accept MP1 and reject one of the block bids.
- t=3,4:
    - Still 5 MW residual ATC, but no regular bids.

This block-acceptance pattern violates **Business Rule 1**, which requires that all single-hour bids priced above AP be fully accepted. Therefore, the configuration **(1,1) is infeasible** and must be excluded from the solution space.

Total welfare:

- MP2: 10×10×4 = 400.
- MP3: 10×10×4 = 400.
- MP1: 0.

Total = **800**.

So both patterns (1,0)/(0,1) and (1,1) produce **800** total welfare. But:

- The pure welfare-maximization problem admits multiple solutions with welfare equal to 800.
- (1,0) and (0,1) keep MP1 fully accepted in MTUs 1–2.
- After enforcing Business Rule 1, the admissible solutions are restricted to: (1,0) and (0,1)
- The configuration **(1,1)** is excluded because it would prevent the full acceptance of a higher-priced single-hour bid.

Therefore, **(y_2, y_3) = (1,0)** or **(0,1)** and (0,0) is dominated.

6.6.4 Auction price computation

MTU	ATC usage (MW)	ATC binding?	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	20 / 25	No	0	**0**
10:15–10:30	20 / 25	No	0	**0**
10:30–10:45	10 / 25	No	0	**0**
10:45–11:00	10 / 25	No	0	**0**

Therefore, the auction price for each MTU is equal to 0.

6.7 Example 6 bis

Time	ATC	MP1	MP2	MP3	MP4
10:00–10:15	30	10 MW @ 20 €/MWh	Block 10 MW @ 10 €/MWh	Block 10 MW @ 10 €/MWh	10 MW @ 5 €/MWh

Time	ATC	MP1	MP2	MP3	MP4
10:15–10:30	30	10 MW @ 20 €/MWh			10 MW @ 5 €/MWh
10:30–10:45	30	—			—
10:45–11:00	30	—			—

6.7.1 Iteration 1: baseline scenario

The block bids are initially rejected. The ATC is fully available in each MTU for single-hour bids.

Subproblem with y_2=y_3=0:

- t=1: accept 10 @20 and 10 @5 → welfare 250, ATC not fully used (capacity slack), λ_1 = 0.
- t=2: accept 10 @20 and 10 @5 → welfare 250, ATC not fully used (capacity slack), λ_2 = 0.
- t=3: → λ_3 = 0.
- t=4: → λ_4 = 0.

So:

- V(0) = 500.
- λ = (0, 0, 0, 0).

Optimality cut from iteration 1:

$$\theta \leq 500$$

The first cut is simply: θ ≤ 500 (no dependence on y_2 and y_3).

Master moves to iteration 2.

6.7.2 Iteration 2: blocks accepted (1, 1)

The master problem becomes bounded by the optimality cut and the feasibility cut:

$$Master = \max(400 \cdot y_2 + 400 \cdot y_3 + \theta)$$

Constraints:

4. $0 \le \theta \le \ 500$
5. $10y_2 + 10y_3 \le \ 25$
6. $y \in \{0; 1\}$

To maximize, master sets θ=500 and y_2=y_3=1, because there's no penalty yet. Objective:

400·1 + 400·1 + 500 = 1,300.

Therefore, the master proposes (1,1).

Subproblem with y_2=y_3=1 and rem. ATC=10:

- t=1: accept 10 @20 → welfare 200, ATC used, λ_1 = 20.
- t=2: accept 10 @20 → welfare 200, ATC used, λ_2 = 20.
- t=3: → λ_3 = 0.
- t=4: → λ_4 = 0.

So:

- V(1,1) = 400.
- λ = (20, 20, 0, 0).

Optimality cut from iteration 2:

$$\theta \le \ 400 - 400(y_2 - 1) - 400(y_3 - 1)$$

$$\theta \le \ 1200 \ - \ 400(y_2 + \ y_3)$$

$$\theta \le 500 \ (optimality\ cut\ from\ iteration\ 1)$$

Now the master is constrained by:

- θ ≤ 500.
- θ ≤ 1200 − 400(y_2 + y_3).

(0,0):

  - θ ≤500 and θ ≤1200−0=1200 → θ≤500
  - Obj = 0 + θ ≤ 500 ⇒ best=500.

(1,0) or (0,1): say (1,0):

- $\theta \leq 500$ and $\theta \leq 800 \Rightarrow \theta \leq 500$
- Obj=400·1 +400·0 + $\theta \leq 400 + 500 = 900 \Rightarrow$ best=900.

(1,1):

- $\theta \leq 500$ and $\theta \leq 400 \Rightarrow \theta \leq 400$
- Obj=400·1+400·1+$\theta \leq 800+400=1200$.

Master still prefers (1,1) with θ=400 ⇒ objective 1,200

Benders selects y*=(1,1) (blocks accepted), θ*=400, total welfare=400 + 400 + 400 = 1200

6.7.3 Introducing business rules

If business rules 1, 2, 3 and 4 were to be taken into consideration by the Benders Decomposition:

6.7.3.1 Pattern A: $(y_2, y_3) = (0,0)$

No blocks accepted.

- Same as iteration 1:
    - t=1: MP1 10 MW @20 and MP4 10 MW @5 → 250
    - t=2: MP1 10 MW @20 and MP4 10 MW @5 → 250
    - t=3,4: 0.
- Total welfare: W_{blocks}(0,0) + V0,0) = 0 + 500 = 500

6.7.3.2 Pattern B: $(y_2, y_3) = (1,0)$ or $(0,1)$ — exactly one block accepted

Remaining ATC per MTU = 20 MW

Subproblem, enforcing business rules:

- t=1,2:
    - Remaining ATC = 20.
    - MP1 wants 10 MW @20.
    - MP4 wants 10 MW @5.
    - Welfare t=1 and t=2:
        - MP1 & MP4: 10×20+10×5 = 250 each.
        - Block: 10×10 = 100 each.

- Total per MTU = 350.

- t=3,4:
    - No regular bids, but block uses 10 MW.
    - Welfare per MTU = block 10×10 = 100.

Total welfare = **900**.

Pattern (0,1) gives the same total welfare by symmetry.

6.7.3.3 Pattern C: (y_2, y_3) = (1,1) — both blocks accepted

Now each block uses 10 MW per MTU, so total block usage per MTU = 20 MW.

Remaining ATC per MTU = 10 MW

Subproblem:

- t=1,2:
    - Remaining ATC = 10.
    - MP1 bid is 10 MW @20.
- t=3,4:
    - Still 10 MW residual ATC, but no regular bids.

Configuration **(1,1) is feasible**.

Total welfare:

- MP2: 10×10×4 = 400.
- MP3: 10×10×4 = 400.
- MP1: 10×10×2=400.

Total = **1200**.

Therefore, **(y_2, y_3) = (1,1).**

6.7.4 Auction price computation

MTU	ATC usage (MW)	Shadow price (€/MWh)	AP (€/MWh)
10:00–10:15	30	20	**10**

MTU	ATC usage (MW)	Shadow price (€/MWh)	AP (€/MWh)
10:15–10:30	30	20	**10**
10:30–10:45	20 / 30	0	**0**
10:45–11:00	20 / 30	0	**0**

Appendix A: Benders Decomposition formulas

Master Objective	$F_{master} = \max\left(\sum_{bb}\sum_{h \in H_b} p_{bb}.d_{bb}.y_{bb} + \theta\right)$
Master Feasibility Cut	$\sum_{bb}\sum_{h \in H_b} p_{bb}.d_{bb}.y_{bb} \leq ATC$
Subproblem Objective **(for a fixed $\bar{y}$, solved for each hour h separately)**	$F_{sub} = V(\bar{y}) = max\left(\sum_{b} p_{b,h}.d_{b,h}\right)$
Subproblem Constraint 1	$\sum_{b} d_{b,h} \leq ATC_h - \sum_{bb}\sum_{h \in H_b} p_{bb}.d_{bb}.\bar{y}_{bb}$
Subproblem Constraint 2	$0 \leq d_{b,h} \leq d_{b,h}^{max}$
Benders Optimality Cut	$\theta \leq V(\bar{y}) - \sum_{h} \lambda_h \left(\sum_{h} d_{bb}.(y_{bb} - \overline{y_{bb}})\right)$